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Professional paper

TELESCOPE POINTING BASED ON INERTIAL MEASUREMENT UNIT

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SUMMARY: In this paper we study the problem of how to determine the coordinates of a point a telescope is directed to on the basis of data obtained from a 9DOF sensor board. On the 9DOF sensor board there are three sensors: the gyroscope, accelerometer and magnetometer. By combining the data from all the three sensors one obtains the Eulerian angles in the system tied to the sensor board. The Eulerian angles are transformed into the horizontal and equatorial coordinates in order to obtain the point the telescope is directed to.

Key words. Methods: observational – Telescopes

1. INTRODUCTION

The idea for this paper has been formed as a result of solving the problem of pointing the Large Refractor towards a given object. This telescope was mounted at the Astronomical Observatory in Belgrade 80 years ago. The limb divisions have been damaged, the optical light path for reading the coordinates has become weak so that nowadays it is difficult to point the telescope towards a desired object. This telescope has no electronics and software directing. Though rather difficult, it is, nevertheless, possible to read the hour angle while the declination reading is almost impossible. In the present paper a simple and cheap solution of telescope pointing towards a desired celestial object is given. From the point of view of the education of young persons and dissemination of astronomy it is very important and useful to use the telescope for these purposes.

To determine the coordinates of a point towards which a telescope is directed, i.e. in order to point a telescope to a desired celestial body, we have tested a sensor board with nine degrees of freedom (9DOF). More precisely, the sensor board used by us consists of three sensors: an accelerometer, gyroscope and magnetometer, which are on a single board (Fig. 1). In commercial use, it is known as Inertial Measurement Units (IMU). The information sent by each of the three sensors in a triaxial coordinate system should be treated within real time and transformed into the Eulerian angles. This is just done by the inertial measurement unit. Here, a relatively cheap IMU with designation "SparkFun 9DOF RAZOR IMU SEN-10736 sensor board" and its applicability for determining the horizontal and equatorial coordinates of objects from the telescope view field will be examined.

2. EULER ANGLES AND ROTATION MATRIX

In order to represent the attitude of an object, the most common way is to use the Eulerian angles (Diebel 2006). They are used to represent the position of a rigid body. Mathematically, the Eulerian angles are formed by rotating one coordinate system relative to the other coordinate system about three axes. Those rotations are made by multiplying the coordinate vectors by rotation matrices Eq. (1), while preserving the length of the vectors. Two coordinate systems of interest are the Earth fixed coordinate system (x, y, z), fixed in inertial space, and the body-fixed coordinate system (X, Y, Z), tied to our IMU and relative to the Earth fixed coordinate system.

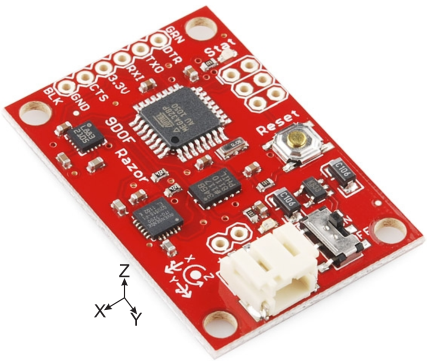

Fig. 1. *SparkFun 9DOF RAZOR IMU SEN-10736 sensor board.*

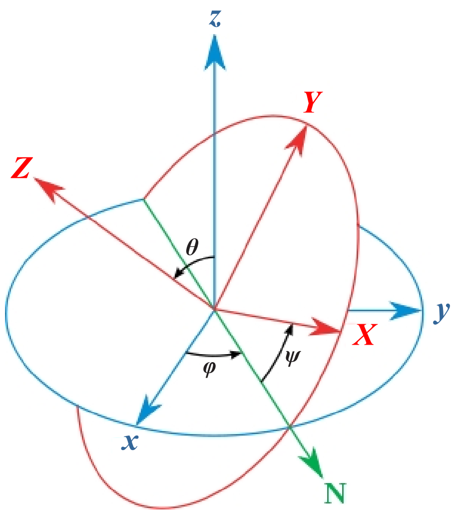

Fig. 2. *Geometrical definition of Euler angles.*

Since the rotation matrix $\boldsymbol{R}$ consists of cosines of angles between the axes of two coordinate systems, it is also known as the direction cosine matrix. In this case, the angles are the Eulerian angles. The additional axis, the line of nodes (N), is necessary for defining the Eulerian angles. This axis represents the intersection of the xy and XY coordinate planes (Fig. 2). Having this in mind, the Eulerian angles can be defined as follows (Andjelić and Stojanović 1965)

- φ is the angle between the x axis and the N axis representing rotation around the z axis; it is known as spin or roll
- θ is the angle between the z axis and the Z axis representing rotation around the N axis; it is known as nutation or pitch
- ψ is the angle between the N axis and the X axis representing rotation around the Z axis; it is known as precession or yaw.

The rotation of vector from body-fixed to the world coordinate system is made through multiplication by the direction cosine matrix (DCM), consisting of cosines of the Eulerian angles (Premerlani and Bizard 2009):

$$\begin{bmatrix} \boldsymbol{R}_x & \boldsymbol{R}_y & \boldsymbol{R}_z \end{bmatrix} . \tag{1}$$

Where:

$$\boldsymbol{R}_x = \begin{bmatrix} \cos\theta\cos\psi \\ \cos\theta\sin\psi \\ -\sin\theta \end{bmatrix} , \tag{2}$$

$$\boldsymbol{R}_y = \begin{bmatrix} \sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi \\ \sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi \\ \sin\varphi\cos\theta \end{bmatrix} , \tag{3}$$

$$\boldsymbol{R}_z = \begin{bmatrix} \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi \\ \cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi \\ \cos\varphi\cos\theta \end{bmatrix} . \tag{4}$$

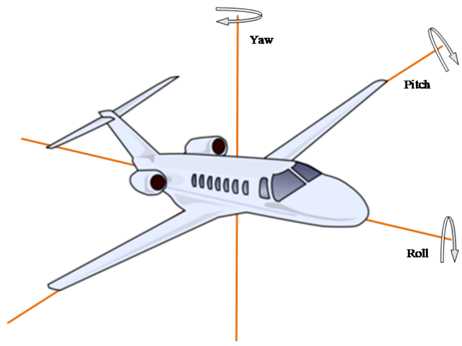

Fig. 3. *Orientation of axes on airplane.*

3. DCM ALGORITHM

The DCM (Direction-Cosine-Matrix) algorithm has been developed for applications on modeling airplanes, helicopters, and other flying machines (Premerlani and Bizard 2009). According to that, the axes of an airplane are referred to as yaw (perpendicular axis), pitch (lateral axis), and roll (longitudinal axis) (Fig. 3).

The DCM algorithm uses the accelerometer, gyroscope, and GPS receiver or magnetometer to obtain the attitude of an object. In our case, we obtained telescope pointing to a horizontal system i.e. altitude and azimuth. The GPS receiver is used in the case of a moving object, but we use it to determine precisely the observer's longitude, latitude and local mean sidereal time. Also, the magnetometer is used when the object is still. The working principle block diagram of the DCM algorithm is shown in Fig. 4 (Premerlani and Bizard 2009).

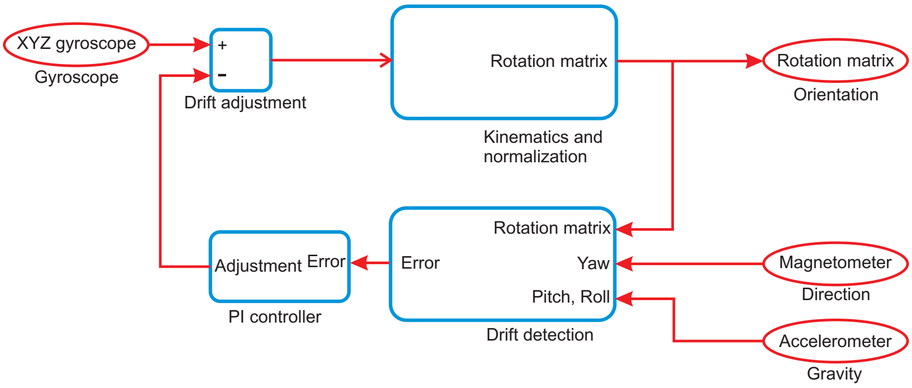

Fig. 4. *Block diagram of DCM algorithm.*

As earlier mentioned, the main part of the sensor device is the SparkFun, 9DOF RAZOR IMU SEN-10736 sensor board[1]. It consists of 3-axes ADXL345 accelerometer, 3-axes ITG-3200 gyroscope, and 3-axes HMC8553L magnetometer. The onboard processor is ATMega 328 @ 8MHz, and can be programmed with Arduino software suite. Together with a Bluetooth module and GPS receiver, this battery powered sensor system is used in our measurements. The data are wirelessly transferred to the computer where application is started to receive and process the data. The whole system scheme is shown in Fig. 5.

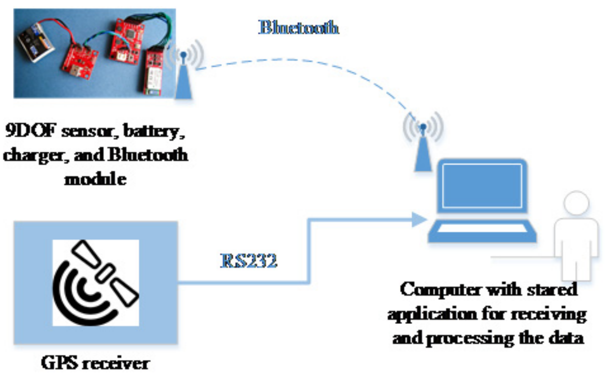

Fig. 5. *The scheme of the system.*

The firmware of the device incorporates the DCM algorithm and can be downloaded from `https://github.com/ptrbrtz/razor-9dof-ahrs`. The outputs of the firmware are Eulerian angles, which are calculated several times per second and then wirelessly transferred to the computer via Bluetooth.

The sensors on SparkFun 9DOF RAZOR IMU have to be calibrated in order to achieve minimum errors and more precise readings. The accelerometer is calibrated by turning the sensor board in every of the nine possible directions of the axes, and then tilted a little until the maximum value is achieved. The gyroscope is calibrated by standing still for ten seconds in order to accumulate the ground movement noise. For the magnetometer calibration, a special processing sketch is used that shows magnetic field disturbances in the environment around the sensor. The sensor has to be moved in every possible direction in order to capture all magnetic disturbances, and it is preferable that they evenly cover the unit sphere. If the data do not evenly cover the unit sphere, there are soft-iron and/or hard-iron disturbances, which have to be eliminated or at least minimized. A more detailed description of the calibrating procedure could be found in Vujičić (2016).

The PC application that receives and processes the data is written in the C# programming language. Its appearance is shown in Fig. 6.

As can be seen from Fig. 6, there are several sections in this application. The Bluetooth connection section is responsible for establishing the Bluetooth connection to the sensor. Under the sensor connection, the user chooses the serial port the Bluetooth module is attached to (via the SPP protocol). In Fig. 6 there is a display of values of Eulerian angles, horizontal coordinates, and equatorial coordinates, azimuth and altitude, and right ascension and declination, respectively. The user can choose between using data from the GPS receiver or getting them from a local storage. Those data are the UTC time, latitude, and longitude. The geographic coordinates are also displayed in a map.

Also, the values of Julian day (JD), Greenwich Mean Sidereal Time (GMST), Local Mean Sidereal Time (LMST), and Hour Angle (HA) are displayed. These values are calculated based on the algorithms from Meeus' book (2009). By clicking on the button Begin Recording, the data from the sensor are captured every 60 seconds and stored in a text file in a format UTC time, azimuth, altitude, right ascension and declination.

[1]Available at `https://www.sparkfun.com/products/10736github/ptrbrtz/razor-9dof-ahrs`

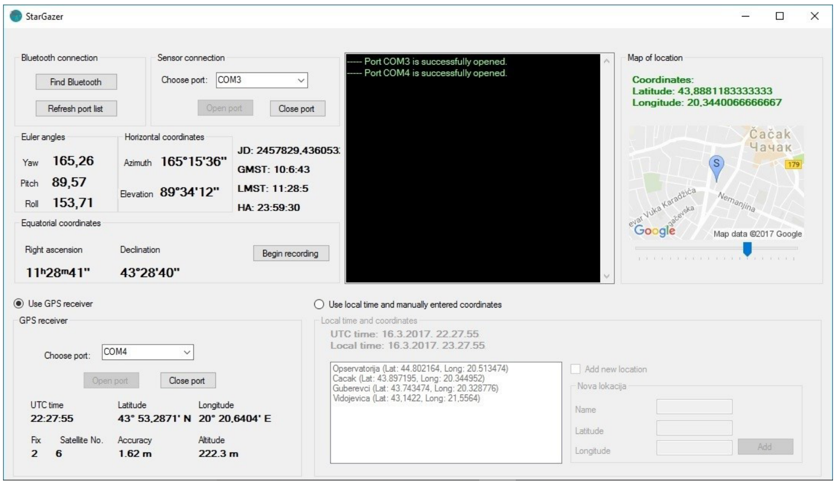

Fig. 6. *The appearance of the PC application.*

4. TELESCOPE POINTING ACCURACY

To determine the telescope positioning precision we place the 9DOF board on the telescope. Since the instrument is without tracking, we point it towards the zenith. After this, we record the data sent by the device every minute within intervals of 50 hours at a location in Čačak and of 11 hours at a location in Guberevci. The testing is carried out at two different places in order to establish if there exists any influence of the local parameters and, if yes, to eliminate this influence. This especially affects the azimuth determination since the 9DOF sensor is very sensitive to the external magnetic field, above all the magnetometer. Thus, our testing took place in Čačak (N 43° 34′ 49″.90 ; E 20° 20′ 41″.80) and Guberevci (N 43° 44′ 36″.51 ; E 20° 19′ 43″.59).

In Fig. 7 the azimuth detected by the 9DOF sensor during the test for Čačak (left) and Guberevci (right) is presented. From the figure it is seen that the azimuth is stable (bearing in mind that the azimuth of zenith is not defined!). At the very beginning of the measuring the scatter is within limits of 20 degrees, with a trend to become twice smaller, in both places. As to the altitude, the situation is somewhat different (Fig. 8). In fact, the scatter during testing was constant, less than 0.5 degrees, but a constant drift towards lower altitudes is noticeable (higher zenith distances). The cause is, most likely, in a drift present in the 9DOF board, but which could be decreased by use of a Kalman filter. Unfortunately, since a cheap board of only 2 KB of memory was used, it was not possible to implement a Kalman filter in the firmware itself.

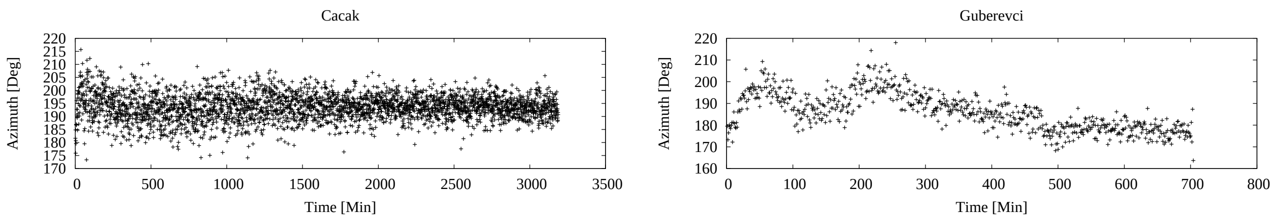

Fig. 7. *Azimuth detected by the 9DOF sensor in Čačak (left) and Guberevci (right).*

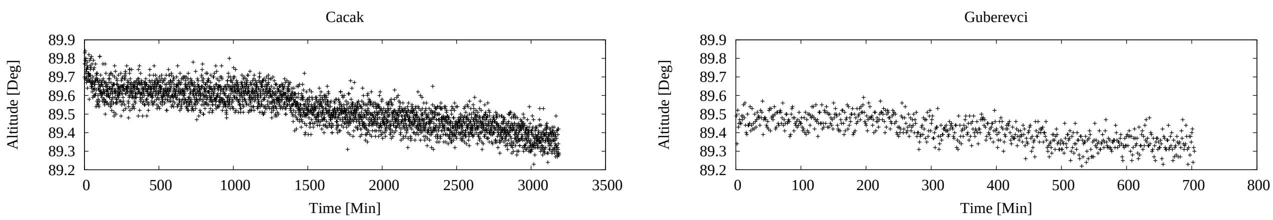

Fig. 8. *Altitude detected by the 9DOF sensor in Čačak (left) and Guberevci (right).*

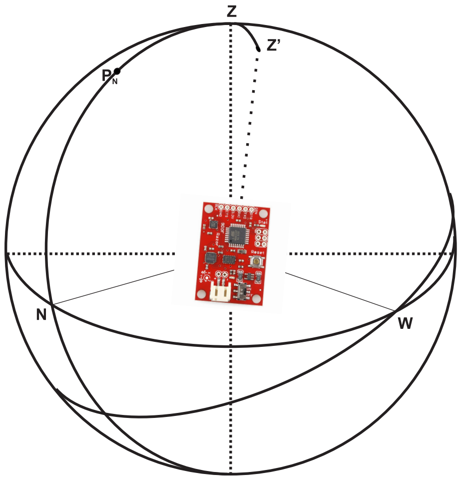

Fig. 9. *Zenith misalignment of the 9DOF board attached to telescope.*

4.1. Determination of confidence ellipse

Since the telescope, as well as the 9DOF board, was not ideally pointed towards the zenith, i.e. it was inclined by a small angle and also rotated by an angle in azimuth, in fact rather than towards the zenith Z it was directed towards a nearby point Z' (Fig. 9). In order to determine the coordinates of the point Z' we use, at first, the rectangular coordinates in the horizontal coordinate system:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{A,h} = \begin{bmatrix} \cos h \cos A \\ \cos h \sin A \\ \sin h \end{bmatrix} \tag{5}$$

where A is the azimuth and h is altitude, and then, in view of the scatter of points about Z' (Fig. 10), we look for the ellipse of minimum area in the plane xy requiring it to include the maximum number of points. In other words, we fit a curve of the second order:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\,, \tag{6}$$

provided that it is an ellipse, i.e. $B^2 - 4AC < 0$. Instead of dealing with all available points, we select only those along the rim which form a convex hull so that all other points are inside it (Fig. 10). To this convex hull we apply the algorithm proposed by Fitzgibbon at al. (1999). In this way we find the ellipse, i.e. we obtain the coefficients A, B, C, D, E and F. The lengths of the semiaxes a and b, the ellipse centre x_0 and y_0, and the angle of ellipse rotation ϑ are determined from the following relations:

$$a = \sqrt{\frac{2(AF^2 + CD^2 + FB^2 - BDE - 2ACF)}{(B^2 - 4AC)\left(\sqrt{(A-C)^2 + B^2} - (A+C)\right)}}\,, \tag{7}$$

$$b = \sqrt{\frac{2(AF^2 + CD^2 + FB^2 - BDE - 2ACF)}{(B^2 - 4AC)\left(-\sqrt{(A-C)^2 + B^2} - (A+C)\right)}}\,, \tag{8}$$

$$x_0 = \frac{CD - BE}{B^2 - 4AC}\,, \tag{9}$$

$$y_0 = \frac{AE - BD}{B^2 - 4AC}\,, \tag{10}$$

$$\vartheta = \frac{1}{2}\arctan\left(\frac{B}{A-C}\right)\,. \tag{11}$$

The ellipse centre (x_0, y_0) is in fact the point Z', whereas the lengths of the semiaxes determine the errors of telescope (board) positioning and ϑ is the angle by which the ellipse is rotated in the azimuthal plane. In Table 1, the obtained results for both places are given, but instead of the coordinates (x_0, y_0), using the inverse relations Eq. (5), we give the azimuth and elevation.

The large value of the standard deviation used to determine the azimuth is a consequence of azimuth undefining at the zenith. On the other hand, the azimuthal angle on the board itself is determined by the magnetometer, which is very sensitive to the presence of the external magnetic field, even to the proximity of metal objects. The other coordinate, altitude, is well determined and its standard deviation is of the order of 0.1 degrees.

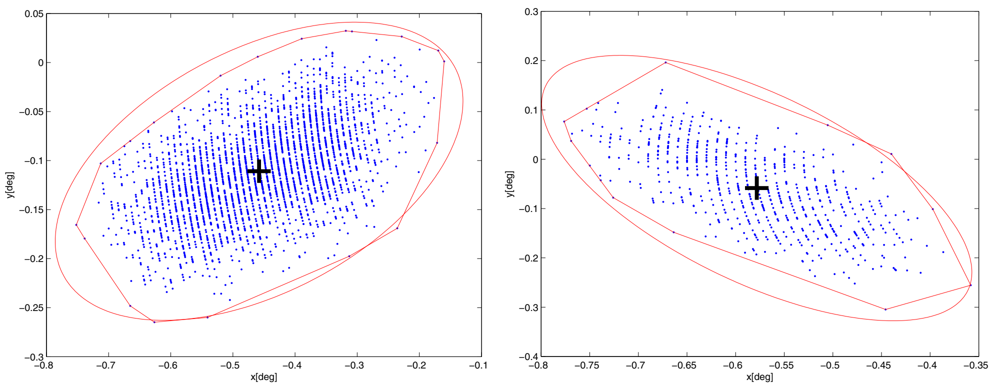

Fig. 10. *The zenith missalignment for Čačak (left) and Guberevci (right). The sign + is the centre of ellipse, i.e. the position of Z'.*

Table 1. Horizontal coordinates of the point Z' with their standard deviations, semiaxis major a and semiaxis minor b, and angle ϑ of the ellipse rotation.

Location	Coordinate of Z'				a	b	ϑ
	Azimuth	σ_{Az}	Altitude	σ_{Alt}			
Čačak	$193^{\circ}\!.7$	$6^{\circ}\!.1$	$89^{\circ}\!.53$	$0^{\circ}\!.10$	$18'\!.53$	$8'\!.14$	$12^{\circ}\!.77$
Guberevci	$185^{\circ}\!.5$	$7^{\circ}\!.9$	$89^{\circ}\!.42$	$0^{\circ}\!.08$	$14'\!.03$	$14'\!.59$	$144^{\circ}\!.39$

5. CONCLUSION

In this paper a simple and cheap solution of telescope pointing towards a desired celestial object is given in the case when there exists the problem of reading the coordinates of its positioning. Of course, this concerns old instruments which have no electronics and software directing. For this purpose the sensor board 9DOF and GPS receiver can serve. The Eulerian angles in the system tied to the sensor board are obtained and then transformed, firstly into the horizontal coordinates, afterwards into the equatorial ones by applying a simple algorithm, i.e. we obtain the coordinates of the point the telescope is directed to. Of course, our task is inverse. Since we know the celestial equatorial coordinates of the object foreseen to be observed, we transform them into the horizontal ones. The telescope can be directed towards a celestial object by moving it and reading the coordinates on the display.

In order to estimate the stability of work of the 9DOF sensor quantitatively we direct the telescope towards the zenith and record the coordinates within an interval of few tens of hours. The estimated errors confirm the stability of the 9DOF sensor board, but also indicate a relatively more precise altitude determination, whereas the azimuth determination error is significantly higher. The cause is, certainly, the magnetometer. However, this can be solved, for instance, by correcting the non-linearity of the magnetometer itself, or by using other algorithms for unifying the data from the sensor, which would yield more stable Eulerian angles. However, in our opinion, even such a cheap sensor can be used for the purpose of telescope positioning.

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REFERENCES

Andjelić, T. and Stojanović, R.: 1965, *Racionalna mehanika*, Zavod za izdavanje udžbenika Socijalističke Republike Srbije, Beograd.

Diebel, J.: 2006. Representing Attitude: Euler Angles, Unit Quaternions, and Rotation Vectors, Stanford University.

Fitzgibbon, A. W., Pilu, M. and Fisher, R. B: 1999, *IEEE Trans. PAMI*, **21**, 476.

Meeus, J.: 1998, Astronomical algorithms, Willmann-Bell, Richmond, Virginia.

Premerlani, W. and Bizard, P.: 2009, Direction Cosine Matrix IMU: Theory, Internal report.

Vujičić, D.: 2016, Primena senzora sa devet stepeni slobode za odredjivanje položaja teleskopa, Master rad, Univerzitet u Beogradu.

ПОЗИЦИОНИРАЊЕ ТЕЛЕСКОПА ПОМОЋУ ИНЕРЦИЈАЛНЕ МЕРНЕ ЈЕДИНИЦЕ

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Стручни чланак

У овом раду се бавимо проблематиком одређивања координата тачке у коју је телескоп уперен, на основу података добијених са инерцијалне мерне јединице тј. 9DOF сензорске плочице. На 9DOF сензорској плочици налазе се 3 сензора: жироскоп, акцелерометар и магнетометар, а комбинацијом података са сва три сензора се добијају Ојлерови углови у систему везаном за сензорску плочицу. Ове Ојлерове углове смо трансформисали у хоризонтске и екваторске координате да бисмо добили тачку у коју је телескоп уперен.