

## ORBIT LIMITED THEORY IN THE SOLAR WIND - $\kappa$ DISTRIBUTIONS

M. M. Martinović<sup>1,2,3</sup>

<sup>1</sup>*LESIA, Observatoire de Paris  
5, place Jules Janssen, 92195 Meudon, France*

<sup>2</sup>*Department of Astronomy, Faculty of Mathematics, University of Belgrade  
Studentski trg 16, 11000 Belgrade, Serbia*

<sup>3</sup>*IHS Techno-experts d.o.o. - Research and Development Center  
Batajnički drum 23, 11080 Belgrade, Serbia*

E-mail: *mihailo.martinovic@obspm.fr*

(Received: February 20, 2016; Accepted: April 15, 2016)

**SUMMARY:** When a solid object is immersed into ionized gas it gets brought to a certain value of electrostatic potential and surrounded by a space charge region called ‘plasma sheath’. Through this region, particles are attracted or repelled from the surface of the charge collecting object. For collisionless plasma, this process is described by the so-called orbit limited theory, which explains how the collection of particles is determined by the collector geometry and plasma velocity distribution function (VDF). In this article, we provide explicit orbit-limited currents expressions for generalized Lorentzian ( $\kappa$ ) distributions. This work is useful to describe the charging processes of objects in non-collisional plasmas like the solar wind, where the electrons VDF is often observed to exhibit quasi power-law populations of suprathermal particles. It is found that these ‘suprathermals’ considerably increase the charge collection. Since the surface charging process that determines the value of electrostatic potential is also affected by the plasma VDF, calculation of the collector potential in the solar wind is described along with some quantitative predictions.

**Key words.** plasmas – Sun: solar wind

### 1. INTRODUCTION

When an object is immersed into the plasma it causes electrostatic disturbances in the surrounding area. The reason of these disturbances that do not exist in neutral gases relies on collection by the object’s (afterward also called collector) surface of charged particles from the plasma. Since the light plasma electrons are faster than the ions, they will more frequently collide with the surface of the col-

lector. This way the surface gets charged and a ‘plasma sheath’ is formed around the object (Langmuir 1923). The sheath size is of the order of the plasma Debye length  $L_D = \sqrt{\epsilon_0 k_b T / qn}$  (where  $n$ ,  $T$  and  $q$  are density, kinetic temperature and charge of particles, while  $\epsilon_0$  and  $k_b$  stand for dielectric permittivity of vacuum and the Boltzmann constant) and it contains a net amount of charge in order to balance the potential of the collector surface. If there are no additional processes responsible for the elec-

tron emission (photoelectric effect, secondary emission, sputtering...) then the potential is expected to be negative. On the other hand, photoelectrons ejected from the illuminated surface are dominant in the sheath around objects in the solar wind, so the potential becomes positive in order to prevent loss of too many electrons.

In the pioneer work on this topic Mott-Smith and Langmuir (1926) first established a theory of orbit limited motion of particles, calculating the incoming particle flux on the collector surface for multiple geometries and plasma velocity distribution functions (VDFs). This work has been extended by many authors. For laboratory plasmas the theory has a lot of limitations, mostly due to very short Debye lengths (Allen 1992, Annaratone et al. 1992). On the other hand, in probe design (Laframboise and Parker 1973) and space plasmas it has found a great number of applications, especially in determining spacecraft floating potentials (see e.g. Scudder et al. 2000, Kellogg et al. 2009), while the quantitative description of suprathermal particles is important for interpretation of electric field fluctuations measurements (Martinović et al. 2016) and charging of dust particles (Meyer-Vernet 1982, Rosenberg and Mendis 1992, Mishra et al. 2013) in the solar wind.

None of the VDFs which have been considered by previous authors took accurately into account the effect of suprathermal particles in the surface charging process. However, non-maxwellian distributions exhibiting large suprathermal tails are ubiquitous in several non-collisional plasma environments and, in particular, in the solar wind (Maksimović et al. 2005, Štverak et al. 2009).

VDFs observed by particle analyzers in the solar wind frequently exhibit a power-law behaviour at large energies and are therefore conveniently modeled by  $\kappa$  distributions (Maksimović et al. 1997). Commonly used models separate three fractions of particles - the core, halo and strahl. The bi-Maxwellian core contains thermal particles, while halo is described by the ‘bi-kappa’ function, both being defined by parallel and perpendicular temperatures with respect to the magnetic field direction. The strahl is defined as a beam of particles moving outward from the Sun and is also described by the bi-kappa distribution, shifted by certain mean velocity from the first two components (Štverak et al. 2009). This kind of the VDF persists in the solar wind as the plasma moves away from the Sun (Zouganelis et al. 2004).

Presence of these fast non-thermal particles modifies both the value of the surface potential and the amount of particles collected by the charged surface of a certain potential. The orbit limited theory quantitatively describes both of these effects, and is given in Section 2 for the case of  $\kappa$  distributions. The theory presented below can be observed as a generalization of the method given by Laframboise and Parker 1973 to non-thermal plasma. Some applications to the solar wind are discussed in Section 3.

## 2. ORBIT LIMITED THEORY FOR $\kappa$ DISTRIBUTIONS

### 2.1. Definition and characteristics of $\kappa$ distributions

This kind of the VDF is actually a generalized Lorentzian (Scudder 1992) power law distribution with a higher percentage of suprathermal particles compared to a classic Maxwellian and is defined as

$$f(v) = \frac{\Gamma(\kappa + 1)}{(\pi\kappa)^{3/2} v_{0\kappa}^3 \Gamma(\kappa - 1/2)} \frac{1}{\left(1 + \frac{v^2}{\kappa v_{0\kappa}^2}\right)^{\kappa+1}}, \quad (1)$$

where  $\Gamma(x)$  denotes the gamma function and  $v_{0\kappa}$  is the thermal speed related to the kinetic temperature  $T$  as

$$v_{0\kappa} = \left(\frac{2\kappa - 3}{\kappa} \frac{k_b T}{m}\right)^{0.5}, \quad (2)$$

with  $m$  being the particle mass. The VDF is defined for  $\kappa > 1.5$  and for  $\kappa \rightarrow \infty$  it reduces to the Maxwellian distribution. This property is important for the solar wind since the value of the  $\kappa$  index can be used as a measure of ‘nonthermality’ of the plasma (low values of  $\kappa$  assume significant portion of suprathermal particles while for  $\kappa \geq 10$  the VDF is so close to the Maxwellian that it can be considered thermal). Further on, it is convenient to define the average velocity as the first moment of the 3D distribution (Chateau and Meyer-Vernet 1991)

$$\langle v \rangle_{\kappa} = 2 \sqrt{\frac{\kappa}{\pi}} \frac{\Gamma(\kappa - 1)}{\Gamma(\kappa - 1/2)} v_{0\kappa}, \quad (3)$$

which converges to the Maxwellian value of  $\langle v \rangle = \sqrt{8k_b T / \pi m}$  for very large values of the  $\kappa$  index.

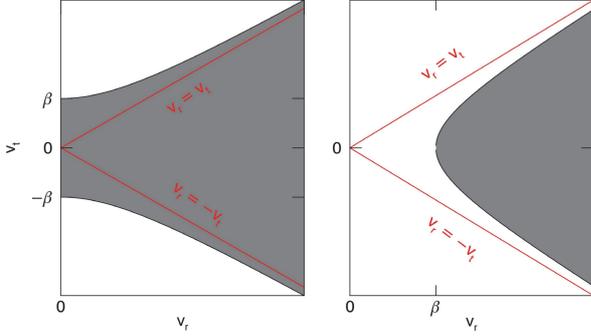
### 2.2. Basics of the model

Since the solar wind plasma we observe here is not Maxwellian and has extremely long mean free paths ( $\sim 1$  AU) it means that collisions can be neglected. We will assume that the radius of the collector  $r_c$  is much smaller than the particles’ mean free path and that every particle has its own trajectory that comes from the ‘infinity’ towards the collector. Also, for cylindrical and spherical collectors, the plasma local Debye length (and the size of the plasma sheath) is required to be very large compared to the radius of the collector. Since  $L_D \approx 5 - 10$  m at distances  $0.3 - 1$  AU from the Sun, both of these conditions are satisfied in the solar wind.

We observe a particle moving with the velocity  $\vec{v}$  in the electrostatic potential  $\phi(\vec{r})$  and note  $v_r$  and  $v_t$  to be radial and tangential components of the particle velocity vector with respect to the collector surface. It can be shown directly from the

laws of conservation of angular momentum and energy (Mott-Smith and Langmuir 1926) that a particle, in order to be able to reach the collector, needs to satisfy following conditions

$$v_r^2 + v_t^2 + \frac{2q\phi}{m} > 0, v_r > 0. \quad (4)$$



**Fig. 1.** Regions in velocity space used for integration of particles for attracting (left) and repelling (right) collector potential. Substitution  $\beta = \sqrt{2|q\phi|/m}$  is made for clarity.

Solutions of Eq. (4) define two hyperbolas with semi-axes  $\pm\sqrt{2|q\phi|/m}$ . Results for both  $q\phi < 0$  (attracting potential) and  $q\phi > 0$  (repelling potential) are given in Fig. 1. Flux of particles that reach the surface are calculated by integrating the VDF over the shaded surfaces. This method is analogous to one used by Laframboise and Parker (1973).

### 2.3. Flux corrections

Now we deal with corrections to the flux of incoming particles per unit surface area of the charged collector. If the collector which is immersed into the plasma is not charged (has zero potential) then the flux of incoming particles will be

$$I_0 = \frac{n \langle v \rangle}{4}, \quad (5)$$

which is a well known result of classical thermodynamics, not depending on the VDF. On the other hand, this result will be modified for a finite value of the potential. As seen below, this change depends on both the collector geometry and VDF of particles.

#### 2.3.1. Attracting potential ( $q\phi < 0$ )

If the potential is attractive (has the opposite sign of particles' charge) then it will increase the incoming particle flux. The exact correction of particle flux for Maxwellian VDF has been calculated by Laframboise and Parker (1973) for three different cases - plain, cylindrical and spherical geometry. Here, we extend those results for  $\kappa$  distributions, showing that a higher portion of suprathermals will make the plasma more sensitive to the potential of the collector.

**Plain geometry** produces one-dimensional potential well. We will assume that the object surface is infinite in the  $yz$ -frame and potential  $\phi$  is dispersed over the  $x$ -axis. For Maxwellian distribution we have

$$I = n\pi^{-3/2}v_0^{-3}e^\eta \int_{\sqrt{\frac{-2q\phi}{m}}}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z v_x e^{-v_x^2/v_0^2} e^{-v_y^2/v_0^2} e^{-v_z^2/v_0^2} = I_0, \quad (6)$$

with  $v_0 = \sqrt{2k_bT/m}$  and  $\eta = -q\phi/k_bT$ . Analogously, for  $\kappa$  distributions we have

$$I_\kappa = n \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)\pi^{3/2}\kappa^{3/2}v_{0\kappa}^3} \int_{\sqrt{\frac{-2q\phi}{m}}}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \frac{v_x}{\left(1 + \frac{v_x^2 + v_y^2 + v_z^2 + \frac{2q\phi}{m}}{\kappa v_{0\kappa}^2}\right)^{\kappa+1}}. \quad (7)$$

From here we make substitutions  $x = v_x\kappa^{-1/2}v_{0\kappa}^{-1}$ ,  $y = v_y\kappa^{-1/2}v_{0\kappa}^{-1}$  and  $z = v_z\kappa^{-1/2}v_{0\kappa}^{-1}$ . Putting these into Eq. (7) we obtain

$$I_\kappa = n \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)} \frac{\kappa^{1/2}v_{0\kappa}}{\pi^{3/2}} \int_{\sqrt{\eta_\kappa}}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz \frac{x}{\left(1 + x^2 + y^2 + z^2 - \eta_\kappa\right)^{\kappa+1}}, \quad (8)$$

where

$$\eta_\kappa = -\frac{2}{2\kappa - 3} \frac{q\phi}{k_bT}. \quad (9)$$

The integral in Eq. (8) is equal to  $\pi/(2\kappa(\kappa - 1))$  and is not dependent on  $\phi$ . Replacing Eq. (3) into Eq. (7) we obtain

$$I_\kappa = I_{0\kappa} = \frac{n \langle v \rangle_\kappa}{4} \quad (10)$$

concluding that there is no change to the particle flux with potential in the plane geometry.

**Cylindrical geometry.** In this geometry it is convenient to use a cylindrical coordinate system with  $z$ -axis parallel to the central axis of the collector and angle  $\theta$  as the angle between the velocity of a particle and the line which is orthogonal to the  $z$ -axis. We start from Eq. (6) in Laframboise and Parker (1973) for the flux per unit surface

$$I = n\pi^{-3/2}v_0^{-3}e^\eta \int_{\sqrt{\frac{-2q\phi}{m}}}^{\infty} dv_r \int_{-\pi/2}^{\pi/2} d\theta \int_{-\infty}^{\infty} dv_z e^{-v_r^2/v_0^2} e^{-v_z^2/v_0^2} v_r^2 \cos\theta, \quad (11)$$

for Maxwellian distribution, which gives the well-known result

$$I = I_0[2\pi^{-1/2}\eta^{1/2} + e^\eta \operatorname{erfc}(\eta^{1/2})] \quad (12)$$

where  $\operatorname{erfc}(x)$  is the complementary error function. Now we use the same approach for the  $\kappa$  distribution by rewriting Eq. (11)

$$I_\kappa = n \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)\pi^{3/2}\kappa^{3/2}v_{0\kappa}^3} \int_{\sqrt{\frac{-2q\phi}{m}}}^{\infty} dv_r \int_{-\pi/2}^{\pi/2} d\theta \int_{-\infty}^{\infty} dv_z \frac{v^2 \cos\theta}{\left(1 + \frac{v_r^2 + \frac{2q\phi}{m} + v_z^2}{\kappa v_{0\kappa}^2}\right)^{\kappa+1}}. \quad (13)$$

From here we make substitutions  $x = v_r \kappa^{-1/2} v_{0\kappa}^{-1}$  and  $y = v_z \kappa^{-1/2} v_{0\kappa}^{-1}$  and obtain

$$I_\kappa = I_{0\kappa} \frac{4\Gamma(\kappa+1/2)}{\pi^{1/2}\Gamma(\kappa-1)} \int_{\sqrt{\eta_\kappa}}^{\infty} \frac{x^2 dx}{(1+x^2-\eta_\kappa)^{\kappa+1/2}}. \quad (14)$$

For the zero potential we have

$$\int_0^{\infty} \frac{x^2 dx}{(1+x^2)^{\kappa+1/2}} = \frac{\pi^{1/2}\Gamma(\kappa-1)}{4\Gamma(\kappa+1/2)}, \quad (15)$$

and, consequently, we have a trivial result  $I_\kappa(\phi=0) = I_{0\kappa}$ .

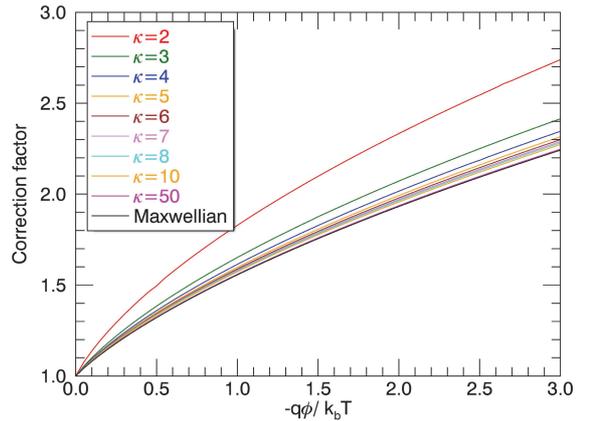
In the general case (the non-zero potential), Eq. (14) can be represented in terms of regularized hypergeometric function  ${}_2\bar{F}_1$  as

$$I_\kappa = -2I_{0\kappa}\pi^{-1/2}\Gamma(\kappa+1/2)\eta_\kappa^{\kappa-1} {}_2\bar{F}_1(\kappa-1, \kappa+1/2, \kappa, 1-\eta_\kappa). \quad (16)$$

For different integer values of  $\kappa$  the general solution is given in Table 1 and Fig. 2. It is clearly visible that increase of the flux is not negligible if the Maxwellian is replaced with the  $\kappa$  VDF.

**Table 1.** Expressions for correction of the particle flux due to attracting potential of a cylindrical collector for multiple values of  $\kappa$  index.

$\kappa$	$I_\kappa/I_{0\kappa}$
2	$\frac{1-(2\eta)^{3/2}}{1-2\eta}$
3	$\frac{9+6^{1/2}\eta^{3/2}(2\eta-5)}{(3-2\eta)^2}$
4	$\frac{-500+10^{1/2}\eta^{3/2}(175+12\eta(\eta-7))}{4(2\eta-5)^3}$
5	$\frac{19208+14^{1/2}\eta^{3/2}(-5145+2\eta(1323+10\eta(2\eta-27)))}{8(7-2\eta)^4}$



**Fig. 2.** Particle flux correction factor in cylindrical geometry due to attracting potential for different values of  $\kappa$  index. Black line is the result given by Mott-Smith and Langmuir (1926), and Laframboise and Parker (1973) for Maxwellian electrons. See the electronic edition of the *Journal* for a color version of this figure.

**Spherical geometry.** In this case we use spherical coordinates with potential  $\phi$  in the  $x$ -direction and  $v_x = v \cos \theta$ . We obtain for Maxwellian distribution

$$I = n\pi^{-3/2}v_0^{-3}e^\eta \int_{\sqrt{\frac{-2q\phi}{m}}}^{\infty} dv \int_0^{\pi/2} d\theta \int_0^{2\pi} d\psi e^{-v^2/v_0^2} v \cos \theta v^2 \sin \theta, \quad (17)$$

resulting with  $I = I_0(1 + \eta)$ . On the other hand, the same method for  $\kappa$  distribution gives

$$I_\kappa = n \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)\pi^{3/2}\kappa^{3/2}v_{0\kappa}^3} \int_{\sqrt{\frac{-2q\phi}{m}}}^{\infty} dv \int_0^{\pi/2} d\theta \int_0^{2\pi} d\psi \frac{v^3 \sin \theta \cos \theta}{\left(1 + \frac{v^2 + \frac{2q\phi}{m}}{\kappa v_{0\kappa}^2}\right)^{\kappa+1}}. \quad (18)$$

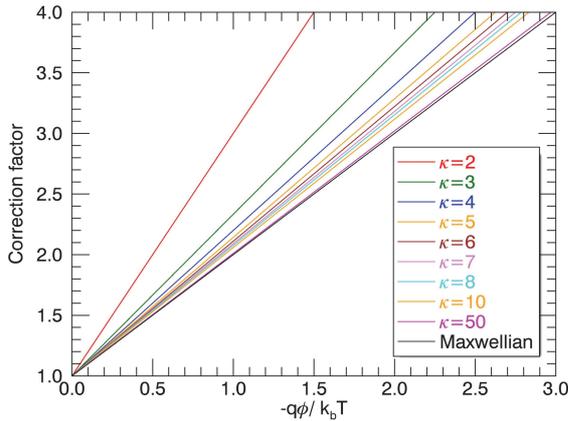
Substituting  $x = v\kappa^{-1/2}v_{0\kappa}^{-1}$  we obtain

$$I_\kappa = I_{0\kappa} 2\kappa(\kappa - 1) \int_{\sqrt{\eta_\kappa}}^{\infty} \frac{x^3 dx}{(1 + x^2 - \eta_\kappa)^{\kappa+1}}, \quad (19)$$

which is solved to give

$$I_\kappa = I_{0\kappa} \left(1 + \frac{2\kappa - 2}{2\kappa - 3}\eta\right). \quad (20)$$

The solution is given in Fig. 3, clearly noting that spherical collectors are more sensitive to the variation of the VDF than cylindrical ones.



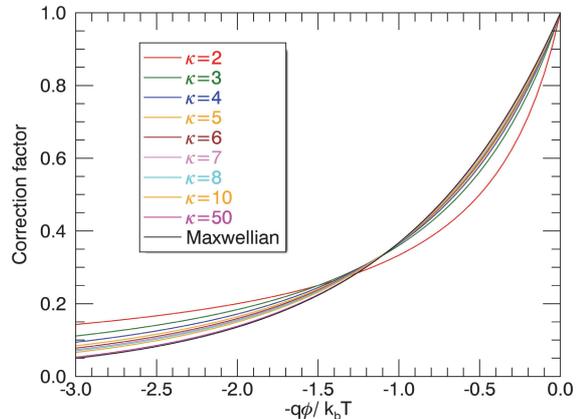
**Fig. 3.** Particle flux correction factor in spherical geometry due to attracting potential for different values of the  $\kappa$  index. Black line is the result of Mott-Smith and Langmuir (1926), and Laframboise and Parker (1973) for Maxwellian electrons. See the electronic edition of the Journal for a color version of this figure.

### 2.3.2. Repelling potential ( $q\phi > 0$ )

All the above calculations assume positive  $\eta$  and  $\eta_\kappa$ , meaning that the potential is attractive. If this is not the case then we deal with the repelling potential which will effectively reduce the incoming particle flux. To calculate this flux, we make use of the same condition given in Eq. (4), integrating along the surface given in Fig. 1 (right). This effectively means that, when  $\eta_\kappa$  is negative, the lower limit of integration in Eqs. (8), (14) and (19) can't go below zero. All three of these equations give the same result for the repelling potential

$$I_\kappa = I_{0\kappa}(1 - \eta_\kappa)^{1-\kappa}. \quad (21)$$

For very large values of  $\kappa$ , Eq. (21) converges to the well-known Maxwellian limit  $e^\eta$ . The result is given in Fig. 4.



**Fig. 4.** Particle flux correction factor due to repelling potential for different values of the  $\kappa$  index. Black line is the result given by Mott-Smith and Langmuir (1926), and Laframboise and Parker (1973) for Maxwellian electrons. See the electronic edition of the Journal for a color version of this figure.

### 2.4. Validity of theory for non-ideal collector shapes

If the collector is not an ideal cylinder or sphere than it is necessary to acknowledge limits for usage of the theory given above. As noted in Section 2.2, the 'orbit limited criterion' for the results

above to be valid is that all particles arrive to the collector from ‘infinity’, so none of the particles can originate from the collector surface. If we assume the opposite - that a particle does originate from the surface of the collector and then returns to it, than such a particle needs to pass through an equipotential point where it will have zero kinetic energy  $E$  (and the maximum distance from the surface) before it starts to ‘descend’ back. Consequently, the criterion is  $E = 0$  for spherical surfaces and  $E_{\perp} = 0$  for cylindrical ones.

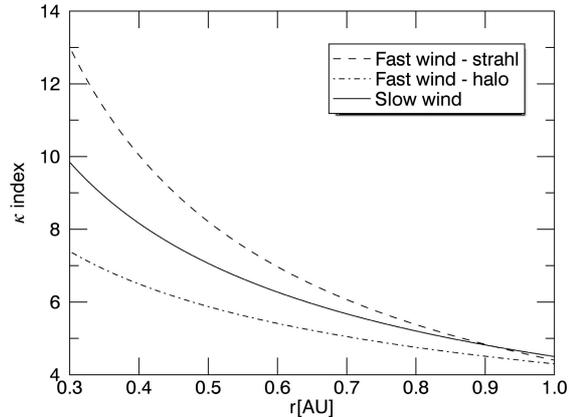
Since these analyses are observing only a single particle without any collective effects, than the results given by Laframboise and Parker (1973) (see Section 3 of that paper) are valid for  $\kappa$  distributions as well as for Maxwellian one. This is the reason why the detailed analysis will not be repeated here, but only the results.

If we have a cylindrical collector than it will not be possible for any of the particles that satisfy conditions from Eq. (4) to pass the surface twice, so we conclude that the ‘orbit limited criterion’ is valid for cylinders for any kind of VDF. Another important point is that, since near the infinite cylinder we always have a zero electric field ( $\vec{E} \rightarrow 0$ ), we can apply the theory not just for circular cylinders but for any kind of a convex cylinder. On the other hand, for spheroids there are limitations for the ratio of the major to minor axis. For a prolate spheroid, the maximum ‘allowed’ ratio is 1.653 and for the oblate ‘Earth-like’ one, the maximum ratio is 2.537. If the spheroid is not completely convex (there are flat surfaces on it), the theory is not applicable. This statement has been confirmed for geometries of several spacecraft (see e.g. Kellogg et al. 2009).

### 3. APPLICATIONS IN THE SOLAR WIND

The velocity distribution for both protons and electrons has been theoretically derived by many authors and measured with particle analyzers during various missions. These results show that the  $\kappa$  index varies with the heliographic distance and solar wind speed. The closest distance covered with spacecraft that give reliable VDF measurements is up to  $\sim 0.3\text{AU}$ . By analyzing the data, it has been proven (see e.g. Štverak et al. 2009) that, beside the Maxwellian core that carries  $\sim 90\%$  of all particles, the  $\kappa$  index decreases with distance from the Sun for both the halo and strahl components. Moreover, it is also evident that  $\kappa$  is lower for the fast solar wind.

Measured values give  $\kappa \approx 10$  at  $0.3\text{AU}$  decreasing with  $\sim r^{0.65 \pm 0.15}$  for the slow wind (for both the halo and strahl),  $\kappa \approx 7$  at  $0.3\text{AU}$  decreasing with  $\sim r^{0.45 \pm 0.1}$  (halo) and  $\kappa \approx 14$  at  $0.3\text{AU}$  decreasing with  $\sim r^{0.9 \pm 0.1}$  (strahl) for the fast wind (Maksimović et al. 2005, Štverak et al. 2009). Approximate values (with  $\sim 20\%$  measurement errors) are given in Fig. 5.



**Fig. 5.** Evolution of the  $\kappa$  index for non-thermal parts of VDFs measured by HELIOS between 0.3 and 1 AU. Results are given in details in the work by Štverak et al. 2009. Uncertainties of the results are less than 25%.

During the missions that cruise towards the Sun, spacecraft need to make the journey from 1AU where they experience the surroundings of the solar wind described by VDF of  $\kappa \approx 4 - 5$  to conditions where the  $\kappa$  index is greater than 10. Beside this, the temperature and density of the solar wind significantly change along the way, which gives growth to the random flux of particles given by Eq. (5). Since the random flux of protons is approximately  $\sqrt{m_p/m_e} \approx 43$  times smaller than the one of the electrons, it can be neglected when we consider charging in the solar wind.

On the other hand, the illuminated surface of the collector will emit electrons due to the photoeffect. The flux of the photoelectrons  $j_{\text{ph}}$  depends only on the sunlight intensity (decreasing with the square of the distance from the Sun) and properties of the surface material. For a given cover material and frequency of incident radiation, the rate at which photoelectrons are ejected from the surface is directly proportional to the intensity of the incident light.

Since the current of secondary electrons can be neglected in the solar wind (Escoubet et al. 1997) we have the equilibrium condition that the current produced by the photoelectrons escaping from the collector potential well and the current of electrons collected from the surrounding plasma  $I_{0\kappa}$  are mutually balanced. If the distribution of emitted photoelectrons is Maxwellian, we have for the current balance equation:

$$S_{\perp} j_{\text{ph}} e^{-\frac{e\phi}{k_b T_{\text{ph}}}} = S I_{0\kappa} \quad (22)$$

where  $S_{\perp}$  and  $S$  are sunlit and total surface of the collector, charge  $q$  is replaced by  $-e$  and  $I_{\kappa}$  is given by Eqs. (10), (16) and (20), depending on the geometry. With  $T_{\text{ph}}$  we mark the photoelectron temperature. This parameter has been measured by many authors and is found to be  $\sim 3$  eV (Henri et al. 2011) for spacecraft covers and  $\sim 2 - 2.7$  eV for standard

BeCu antennas (Scudder et al. 2000, Pedersen et al. 2008, Kellogg et al. 2009). Assuming these parameters, the potential  $\phi$  of the collector can be found as the solution of Eq. (22). Since the values of  $I_\kappa$  are by an order of magnitude below the values of  $j_{ph}$  in the solar wind, it is clear that potential of the antenna and spacecraft surface will be positive - attracting electrons. A negative potential can appear only in shadowed areas.

**Affection of the strahl.** By the name strahl is presumed an anisotropic beam-like fraction of the VDF. Although participation of the strahl in the total particle population is less than 5% and decreases with distance from the Sun, it carries a significant fraction of the heat flux since the beam is faced away from the Sun along the magnetic field direction (Maksimovic et al. 2005). The strahl is well described by the  $\kappa$  distribution shifted by the mean velocity  $v_{beam}$ . Consequently, in the reference frame moving with the velocity  $v_{beam}$ , it can be described by a standard  $\kappa$  VDF with the mean velocity  $\langle v \rangle_{s0}$  given by Eq. (3). On the other hand, in the spacecraft reference frame (if we consider the magnetic field to be radial in the first approximation) the particle random flux on the sunlit surface originating from the strahl is given as  $I_{0\kappa s} = n_s \langle v \rangle_s / 4$  where  $n_s$  is the number density of particles in the beam and  $\langle v \rangle_s = \langle v \rangle_{s0} + v_{beam}$ . The contribution of this component is additive to the isotropic part.

#### 4. CONCLUSION

Any charged collector object in the plasma will perturb it by attracting particles with the opposite charge and repelling particles with the charge of the same sign. The orbit limited theory, established by Mott-Smith and Langmuir (1926), quantitatively describes this effect, proving that both the collector charging and collection of surrounding particles depend on the collector shape (geometry) and plasma VDF. Laframboise and Parker (1973) applied the theory to the plasma probe design, using a new mathematical model, applicable to space plasmas. In this work, the same approach is used to extend the orbit limited theory to the solar wind plasma described by the  $\kappa$  distributions. These results are of importance since the accurate determination of the spacecraft and antenna potential is needed for improvement of *in situ* measurements in space missions.

By analyzing the plain, cylindrical and spherical geometries it is shown that the correction of the particle flux at the surface of the collector in a space plasma is strongly dependent of the VDF shape. A higher percentage of suprathermals (that corresponds to lower values of the  $\kappa$  index) considerably increases the particle collection, with spheres being more sensitive than cylinders. These results are valid for any kind of convex cylinders, while spheroids have strictly defined maximum level of elongation that must not be breached for the calculations to be trustworthy.

On the other hand, the collector charging process is also defined by the same parameters. Electrostatic potential of the illuminated surface is calculated directly from the balance of the photoelectron and plasma current. Calculation of the surface potential using results presented here can be performed for a spacecraft and its antennas more accurately in comparison with standard calculation that assumes a thermal plasma, since it enables implementation of a more realistic plasma model described by the  $\kappa$  distributions.

*Acknowledgements* – Thanks to Arnaud Zaslavsky, Milan Maksimović and Stevo Šegan for useful advices that improved this paper and also to the reviewer for constructive comments and helpful suggestions on an earlier version of the manuscript. During the work on the paper the author was financially supported by the Ministry of Education and Science of the Republic of Serbia through the project 176002.

#### REFERENCES

- Allan, J. E.: 1992, *Phys. Scripta*, **45**, 497.  
 Annaratone, B. M., Allan, M. W. and Allan, J. E.: 1992, *J. Phys. D*, **45**, 497.  
 Chateau, Y. F. and Meyer-Vernet, N.: 1991, *J. Geophys. Res.*, **96**, 5825.  
 Escoubet, C. P., Pedersen, A., Schmidt, R. and Lindqvist, P. A.: 1997, *J. Geophys. Res.*, **102**, 17595.  
 Henri, P., Meyer-Vernet, N., Briand, C. and Donato, S.: 2011, *Phys. Plasmas*, **18**, 082308.  
 Kellogg, P. J., Goetz, K., Monson S. J., Bale, S. D., Reiner, M. J. and Maksimović, M.: 2009, *J. Geophys. Res.*, **114**, A02107.  
 Laframboise, J. G. and Parker, L. W.: 1973, *Phys. Fluids*, **16**, 629.  
 Langmuir, I.: 1923, *Science*, **58**, 290.  
 Maksimović, M., Pierrard, V. and Lemaire, J. F.: 1997, *Astron. Astrophys.*, **324**, 725.  
 Maksimović, M. et al.: 2005, *J. Geophys. Res.*, **110**, A09104.  
 Martinović, M. M. et al.: 2016, *J. Geophys. Res.*, **121**, 129.  
 Meyer-Vernet, N.: 1982, *Astron. Astrophys.*, **105**, 98.  
 Mishra S. K., Misra S and Sodha, M. S.: 2013, *Eur. Phys. J. D*, **67**, 210.  
 Mott-Smith H. M. and Langmuir, I.: 1926, *Phys. Rev.*, **28**, 727.  
 Pedersen, A. et al.: 2008, *J. Geophys. Res.*, **113**, A07S33.  
 Rosenberg, M. and Mendis D. A.: 1992, *J. Geophys. Res.*, **97**, 14773.  
 Scudder, J. D.: 1992, *Astrophys. J.*, **398**, 299.  
 Scudder, J. D., Cao X. and Mozer F. S.: 2000, *J. Geophys. Res.*, **105**, 21281.  
 Štverak, S., Maksimović, M., Travičnik, P., Marsch, E., Fazakerley A. N. and Scime, E. E.: 2009, *J. Geophys. Res.*, **114**, A05104.  
 Zouganelis, I., Maksimović, M., Meyer-Vernet, N., Lamy, H. and Issautier, K.: 2004, *Astrophys. J.*, **606**, 542.

ТЕОРИЈА ОГРАНИЧЕНОГ ОРБИТАЛНОГ КРЕТАЊА  
У СУНЧЕВОМ ВЕТРУ -  $\kappa$  РАСПОДЕЛА

M. M. Martinović<sup>1,2,3</sup>

<sup>1</sup>*LESIA, Observatoire de Paris  
5, place Jules Janssen, 92195 Meudon, France*

<sup>2</sup>*Department of Astronomy, Faculty of Mathematics, University of Belgrade  
Studentski trg 16, 11000 Belgrade, Serbia*

<sup>3</sup>*IHIS Techno-experts d.o.o. - Research and Development Center  
Batajnički drum 23, 11080 Belgrade, Serbia*

E-mail: *mihailo.martinovic@obspm.fr*

UDK 523.62–726

*Оригинални научни рад*

Када се чврсто тело налази окружено јонизованим гасом оно бива доведено на одређену вредност електростатичког потенцијала и окружено наелектрисаном регионом званом "плазмени плашт". Кроз овај регион честице бивају привучене или одбијене од површине наелектрисаног тела (колектора). За плазму у којој нема судара, овај процес је описан теоријом ограниченог орбиталног кретања, која објашњава на који начин је интензитет сакупљања честица одређен геометријом колектора и функцијом расподеле самих честица. У овом раду дати су експлицитни изрази за орбиталне струје код функција

расподеле облика генерализованог лоренцијана ( $\kappa$  расподеле). Резултати овог рада су корисни у описивању процеса наелектривања објеката у несударним плазмама као што је Сунчев ветар, где мерене функције расподеле садрже нетермалне брзе честице и описане су степеним  $\kappa$  функцијама. Пронађено је да брзе "супратермалне" честице значајно појачавају сакупљање честица на колектору. Како функција расподеле такође утиче и на процес формирања електростатичког потенцијала површине колектора, дат је и поступак израчунавања истог у Сунчевом ветру заједно са неким квантитативним предвиђањима.