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SPIRAL SOLITON SOLUTION FOR DISK GALAXIES

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SUMMARY: In this paper, weakly nonlinear dynamics of spiral galaxies is studied, using reductive perturbation method. One primarely aims at the derivation of possible soliton solution for two dimensional geometry, in the state of marginal stability. In order to use proper coordinate transformation, it was necessary to analyze stability of the linearized system of equations, and to define proper parameter regime. Such parameter regime is in agreement with the observational data, too. The influence of finite-thickness of the galaxy disk on dispersive properties of the system is studied, extending approximate solution of Poisson's equation. For both cases, infinitesimaly thin disk and disk of finite thickness, the same type of NLS equation is derived, but with different coefficients for nonlinear and dispersive terms. This means that corresponding soliton solutions have different properties. By comparing soliton properties with observational data it is possible to control validity of approximation for different geometry of the model.

Key words. Galaxies: spiral – Galaxies: kinematics and dynamics – Methods: analytical

1. INTRODUCTION

Spirals are rather common structures produced in many different systems such as atmospheric flows, some self-catalyzed chemical reactions, a variety of networks (neurons, circuits or ecosystems), and some life forms. The seminal work of Lin and Shu (Bertin 2000) succeeded in producing a spiral solution of a linearized density wave equation. In the present work, we consider a nonlinear dispersive wave model to study nearly collisionless dynamics of the spiral galaxies, using reductive perturbation method (Jaffrey and Taniuti 1964), with the emphasis on possible soliton solutions. Two different geometries of the disk are discussed, and the corresponding solutions of the nonlinear equation are given.

2. GOVERNING EQUATIONS

The density wave model consists of transport equations for the mass density ρ and the momentum $\rho\nu$, together with the Poisson's equation that relates the density to the gravitational potential ϕ. The equilibrium state of the system is described as a rotation with an angular velocity $\Omega(r)$ about z-axis under the balance of centrifugal and gravitational forces in a frame rotating with constant angular velocity Ω_0. Then, the equilibrium velocity is $\nu_{0\varphi} = (\Omega - \Omega_0)r$, where $\Omega^2 r = -\partial\phi_0/\partial r$. The quantities ϕ_0 and ρ_0 are the equilibrium potential and the density, respectively. The dispersive property originates from the coupled Poisson's equation, which is a second-order elliptic partial differential equation.

Case (a): The model of Lin and Shu assumes delta function for the density in z-direction and approximates Poisson's equation by

$$\frac{\partial\phi(r, z = 0)}{\partial r} = \pm 2\pi \mathrm{i} G\sigma, \quad (1)$$

in the vicinity of spiral arms, where σ represents surface mass density (Lin and Shu 1964). Here, the geometry of the model is infinitely thin disk.

Case (b): In this paper we propose more realistic solution, introducing in the z-direction Gaussians instead of delta function, $f(z)$ for potential and $g(z)$ for density. Then, we can approximately express Poisson's equation in dimensionless form as follows:

$$A\nabla_\perp \hat{\phi} + B\hat{\phi} = \hat{\phi}, \quad (2)$$

where $\hat{\phi}, \hat{\rho}$ are two-dimensional (r and φ dependent) potential and density, respectively, $A = -a/(4\pi Gc)$, $B = -b/(4\pi Gc)$ are constants dependent on thickness of the disk L by way of a, b and c given by:

$$a = (1/2L)\int_{-L}^{L} f(z)dz,\ b = (1/2L)\int_{-L}^{L} f''(z)dz,$$

$$c = (1/2L)\int_{-L}^{L} g(z)dz, \quad (3)$$

and $\nabla_\perp^2$ denotes two-dimensional Laplacian in the plane perpendicular to z.

3. NONLINEAR EQUATION WITH SPIRAL SOLITON SOLUTION

Case (a): Let us first examine two-dimensional fluid model of the infinitesimally thindisk galaxy (Lin-Shu approximation). We normalize r and φ by means of the wave length of the carrier wave in the radial direction, $2\pi R/\lambda$, where R is the radial size of the galaxy and $\lambda \gg 1$ is a dimensionless constant resulting from the Lin-Shu derivation; t is normalized by the period of the carrier wave $2\pi/\omega$, ρ by ρ_0, both components of velocity by the phase velocity $\omega R/\lambda$, ϕ by $\omega^2 R^2/\lambda^2$ and G by $\omega^2 R/(2\rho_0\lambda)$. Introducing $\tau = t + \varphi/\Omega$, the set of governing equations will be somewhat reduced. Before making the choice of transformation of coordinates and expansion of variables, it is necessary to discuss parameter regime. Dispersion relation in this case will be (Fig. 1):

$$\omega^2 = \kappa^2 - 2\pi G\rho_0|k|. \quad (4)$$

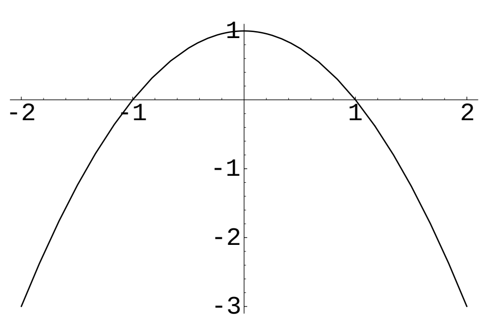

Fig. 1. *Marginal stability curve for the zero thickness fluid model. x axis represents wave number k normalized by critical wave number k_2, and y axis represents Doppler shifted frequency ω^2 normalized by epicyclic frequency κ^2.*

Stability parameter is defined by $k_2 = \kappa^2/(2\pi G\rho_0)$, so that all waves with $k < k_2$ are purely stable. For this regime, dark soliton solution has already been obtained (Kondoh et al. 2000). The problem is that this solution has dark soliton solution with diminishing density, and has no spiral pattern.

Taking into account initial limitation on k, namely $k > k_1$ (where k_1 =max$\{1/r, f'/f\}$, $f = \rho_0(r)$ and prime denotes the derivative with respect to r), we find that observational data suggest $k \approx k_2$. Marginal stability, as introduced above in terms of local dispersion relation, defines a very important condition for the basic state. In fact, if the system is far on the side of instability, then it can be expected from it to be subject to rapidly growing perturbations, which are bound to change the properties of the basic state on a short dynamical time scale. In astrophysical applications, it is often said that violently unstable models are just the wrong choice of basic state (Bertin 2000). The relevant regimes for the galaxy disk must be close to the instability threshold. In this case, a new transformation of variables has to be introduced according to Watanabe (Watanabe 1969), different from the stable case (the reason being that, in marginal stability, frequency goes to zero, so that the group velocity becomes infinite). Starched coordinates and expansion of variables in our case are given as:

$$\begin{aligned} \xi &= \varepsilon(\tau - cr), \quad \eta = \varepsilon^2 r, \\ \rho &= \rho_0 + \sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} \varepsilon^n \rho^{n,m}(\xi,\eta) e^{\mathrm{i}(kr-\omega t)}, \quad (5) \\ \nu_{fi} &= r\Omega + \sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} \varepsilon^n \nu_\varphi^{n,m}(\xi,\eta) e^{\mathrm{i}(kr-\omega t)}. \end{aligned}$$

Substituting (5) into governing equations (the transport equations of mass density and momentum), and using Lin-Shu solution (1) instead of Poisson's equation, we derive the nonlinear equation:

$$\mathrm{i}\frac{\partial\rho^{1,1}}{\partial\eta} + P\frac{\partial^2\rho^{1,1}}{\partial\xi^2} + Q|\rho^{1,1}|^2\rho^{1,1}, \quad (6)$$

in which $P = -k_2/\kappa^2 = -1/2(\partial k/\partial\omega^2) < 0$, and $Q = -(3/2)\kappa^2/(k_2\rho_0^2) < 0$, so that $PQ > 0$. This type of equation has bright soliton solution moving in the ξ direction:

$$\begin{aligned}\rho^{1,1}(\xi,\eta) &= \rho_0 \frac{e^{\mathrm{i}\nu}}{ch(\sqrt{(B/2A)}\rho_0(\xi-2A\eta))}, \\ \psi &= A\left(\frac{B}{2A}\rho_a^2-1\right)\eta+\xi. \end{aligned} \tag{7}$$

Going back to the original coordinates, one obtains the solitary structure solution with enhanced density along the spiral, which explains the observed pattern (see Fig. 2).

Case (b): We extend nonlinear analysis to the more realistic case, taking the finite thickness effect into account by way of the Poisson's equation (2). It will yield for k (marginal stability case), a NLS equation with the coefficients A and B dependent of n ($B/A = n$ that includes information about the thickness). Since these coefficients determine the amplitude, width and velocity of the soliton, a comparison with the structure observed, makes it possible to decide when the finite thickness approximation is necessary for a given galaxy.

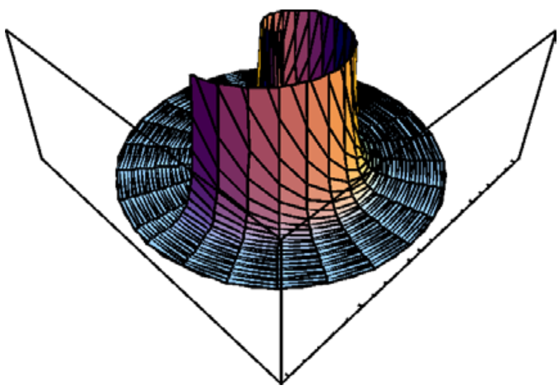

Fig. 2. *Enhanced density along the spiral in 3d; solution of Eq.(4).*

4. CONCLUSION

In this paper we studied weakly nonlinear dynamics of different galaxy models, using reductive perturbation method, with the emphasis on possible soliton solutions. For 2-dimensional model, using Lin-Shu approximation, the NLS equation was derived. Solution is the bright soliton, propagating along the spiral. Having established the solitary solution, we eliminate the main difficulty from the linear theory, that is the problem of searching generators of spiral wave and mechanism that maintains waves on a long time scale (quasy-stationarity assumption). We extended the 2-dimensional analysis for galaxies by solving the Poisson's equation in a different manner and obtaining NLS equation. The latter is with coefficients different for nonlinear and dispersive terms, which means different properties of soliton. Comparing the evaluated soliton properties with the observational data, one can control the validity of approximations used in either of the models.

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СПИРАЛНО СОЛИТОНСКО РЕШЕЊЕ ЗА ДИСКОЛИКЕ ГАЛАКСИЈЕ

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Оригинални научни рад

У овом раду је проучавана слабо нелинеарна динамика спиралних галаксија, употребом редуктивне пертурбационе методе. Основни циљ је одређивање могућег солитонског решења за дводимензиону геометрију, за систем који је у стању граничне стабилности. У циљу употребе одговарајуће трансформације координата неопходно је претходно извршити анализу линеаризованог система једначина и дефинисати одговарајући режим параметара. Режим параметара дефинисан на овај начин у сагласности је са посматрачким подацима. Проучен је утицај коначне дебљине галактичког диска на дисперзивне особине система, решавајући Поасонову једначину у проширеном облику у односу на претходно апроксимативно решење Поасонове једначине које су предложили Лин и Шу. У оба случаја, за бесконачно танак диск, као и за диск коначне дебљине, изведен је исти облик нелинеарне Шредингерове једначине, али са различитим коефицијентима уз нелинеарни и дисперзивни члан. Ово значи да одговарајућа солитонска решења имају различите особине. Поређењем особина солитона са посматрачким подацима могуће је контролисати да ли је коришћена апроксимација за геометрију модела оправдана.