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Preliminary report

A MODIFIED THEORETICAL $\Sigma - D$ RELATION FOR SUPERNOVA REMNANTS: II. THE CASE OF VARIABLE TEMPERATURE WITHIN THE SUPERNOVA REMNANT

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SUMMARY: In this preliminary report a modification of the theoretical $\Sigma - D$ relation for supernova remnants (SNRs) in the adiabatic expansion phase is presented in another way. As in Paper I, this modification is based on the convolution procedure. The relation first derived by Shklovsky is convoluted with the $\Sigma - D$ relation derived in this report for thermal bremsstrahlung radiation. Also, we adopt McKee & Ostriker's model for the interaction between SNRs and the interstellar medium. Kesteven's modified theoretical relation gives the best agreement with the updated "master" empirical $\Sigma - D$ relation derived by Urošević.

Key words. ISM: supernova remnants – radiation mechanisms: thermal – radio continuum: ISM

1. INTRODUCTION

The $\Sigma - D$ relation (the relation between the radio surface brightness Σ and the diameter D of supernova remnants (SNRs)) provides a convenient way to investigate the surface brightness evolution of SNRs. A detailed description of the development of theoretical and empirical $\Sigma - D$ relations is given by Urošević, Duric & Pannuti (this issue, hereafter Paper I).

Shklovsky (1960) derived the theoretical $\Sigma - D$ relation of the form:

$$\Sigma = AD^{-\beta} \tag{1}$$

directly from synchrotron radiation theory. In that theory, the thermal component of the SNR's radiation is neglected, even though it probably does influence the $\Sigma - D$ relation. In this paper it will be shown, in another way, how the thermal component could influence the $\Sigma - D$ relation.

Values for the exponent β in the empirical relations ($\beta \approx 2$) are less than the values expected by theory ($\beta = 3.5$), and perhaps the empirical - theoretical inconsistency can be at least partially explained by the omission of the thermal component. If we derive the $\Sigma - D$ relation by taking into account the thermal radiation from the ionized gas cloud (that is, bremsstrahlung emission from the free electrons moving through the field of the positively charged ions) and in some way associate it with relations derived for the synchrotron mechanism, we may obtain a $\Sigma - D$ relation with a significantly reduced value for β.

As in Paper I, the convolution method will be used to combine these two $\Sigma - D$ relations for different radiation mechanisms. Also, we adopt the model presented by McKee & Ostriker (1977, hereafter M&O) for the interaction between SNRs and the interstellar medium.

2. MODIFICATION OF THE THEORETICAL $\Sigma - D$ RELATION

2.1. $\Sigma - D$ relation for thermal radiation from the ionized gas cloud: The case of variable temperature

Using equations for the volume thermal emissivity and for the radiation intensity of the spherical ionized gas cloud (Eqs. (2) and (3) presented in Paper I), we obtain:

$$I_\nu = \frac{R}{4\pi}\frac{4}{3}\frac{Z^2e^6}{c^3}\frac{N_iN_e}{m^2}\Big(\frac{2m}{\pi kT}\Big)^{\frac{1}{2}}\ln\frac{p_2}{p_1}. \tag{2}$$

We assume that the concentration of particles does not change with distance from the center of the cloud: this assumption is consistent with the M&O model. Since the SNR is assumed to be in the adiabatic phase (*i.e.*, the SNR will adibatically cool while expanding), we start with the adiabatic equation, expressed as:

$$TV^{\gamma-1} = const. \tag{3}$$

In the case of a spherical cloud and assuming $\gamma = \frac{5}{3}$ (*i.e.*, assuming that the gas in the SNR interior behaves like an ideal gas), we obtain the following dependence of temperature with respect to cloud radius:

$$T \propto R^{-2}. \tag{4}$$

Substituting relation (4) into Eq. (2) and assuming that the radiation intensity of the object is constant, the flux density $F_\nu = \int I_\nu d\Omega$ may therefore be expressed as:

$$F_\nu \propto R^3(R^{-2})^{-1/2} = R^4. \tag{5}$$

According to relation (5), we then have:

$$\Sigma_\nu \propto R^2. \tag{6}$$

Since it is common knowledge that SNRs also possess relativistic electrons which emit synchrotron radiation, we may consider these relativistic particles to derive another constraint on the dependence of surface brightness on radius. If the total energy of a particle is much greater than its rest mass, the rest mass may therefore be ignored when considering the particle's total energy. Similar to the case of an ideal gas, if we neglect relativistic corrections for temperatures $T \leq 10^6$K (see Rybicki & Lightman 1979) and set $\gamma = \frac{4}{3}$, we derive the following dependence of surface brightness with respect to cloud radius:

$$\Sigma_\nu \propto R^{1.5}. \tag{7}$$

Therefore, in accordance with the M&O model, relations (6) and (7) give a $\Sigma - D$ relation for thermal emission from SNRs of the following form:

$$\Sigma_\nu \propto D^{1.5\leq-\beta\leq 2.0}. \tag{8}$$

Based on inspection of this relation, we notice that as the size of the SNR increases, the surface brightness also increases: this result is valid for the case of an optically thin medium. For this reason, it is necessary to examine whether the medium is transparent for some frequency (1 GHz) used for the construction of the $\Sigma - D$ relation. We have already presented an extended discussion on this topic in Paper I. The M&O model of the interaction between SNRs and the interstellar medium leads to the conclusion (as presented in Paper I) that the SNR is optically thin at 1 GHz. We note that cold cores may not be optically thin at this frequency, but the very small volume filling factor of these cores permits us to safely neglect them in our analysis.

2.2. $\Sigma - D$ relation for synchrotron radiation and thermal radiation from the ionized gas cloud: The case of variable temperature

If relation (6) is convoluted with Shklovsky's relation ($\beta = 6$), we obtain the following integral form for $\Sigma(t)$:

$$\Sigma(t) \propto \int_0^\infty \frac{R^2}{(t-R)^6} dR. \tag{9}$$

As in Paper I, the integrals are evaluated over the range of $R = 0$ through $R = +\infty$ to describe the expansion of the SNR from very small radii (nearly zero) at the beginning of the explosion through later periods when the SNR reaches enormously large radii (the limiting case is ∞). This integral has the following solution:

$$\Sigma(t) \propto t^{-3}. \tag{10}$$

From this relation, we conclude that by combining the results of Shklovsky's theory with the results obtained in Section 2.1, the relation between surface brightness and diameter may be expressed as follows:

$$\Sigma_\nu \propto D^{-3}. \tag{11}$$

As in Paper I, the introduction of a thermal component to Shklovsky's theory leads to a form of the $\Sigma - D$ relation which is very close to the empirical relation. In the case of a relativistic gas, the function $R^{1.5}$ may be convolved with the function R^{-6} to give the following result:

$$\Sigma(t) \propto \int_0^\infty \frac{R^{1.5}}{(t-R)^6} dR \propto t^{-3.5}, \tag{12}$$

from which we obtain:

$$\Sigma_\nu \propto D^{-3.5}. \tag{13}$$

The theoretical model which assumes a constant shell thickness (Kesteven 1968) yields a $\Sigma - D$ relation in the following form: $\Sigma \propto D^{-4.5}$ (for $\alpha = 0.5$). Under the assumption of a shell model (consistent with the model of M&O), thermal flux from the shell can be expected. In this case, flux from the hot interior may be neglected because the particle concentration is higher in the shell, resulting in a greater efficiency of thermal radiation from the ionized gas cloud (Eq. 2 in Paper I). Relativistic particles in the shell will contribute, thereby introducing the thermal component to the total surface brightness as is shown in relation (7). The convolution integral in this case is:

$$\Sigma(t) \propto \int_0^\infty \frac{R^{1.5}}{(t-R)^{4.5}} dR \propto t^{-2}. \tag{14}$$

Similar to the previous convolution, we obtain:

$$\Sigma_\nu \propto D^{-2}. \tag{15}$$

This relation has a value for β which is closest to the latest empirical $\Sigma - D$ relations (Urošević 2000, 2002, 2003) for the average values of the spectral index $\alpha = 0.5$.

The surface brightness in relation (15) decreases with an inverse square-law dependence as the radius of the SNR increases, giving a solution for the simple spherical expansion of the SNR as the luminosity remains constant. This effect – the independence of luminosity with respect to SNR diameter – has already been detected on the several occasions (e.g. Mills et al. 1984, Stanković, Tešić & Urošević 2003). The relation $\Sigma_\nu \propto \frac{F_\nu}{\theta^2}$ (where θ is angular diameter), combined with $D \propto \theta d$ (where d is distance to the remnant) and $L_\nu \propto F_\nu d^2$ (L_ν represents radio luminosity of the remnant per unit frequency interval), may be transformed into

$$\Sigma_\nu \propto L_\nu D^{-2}. \tag{16}$$

In the case where luminosity is independent of radius, this relation simplifies to relation (15).

3. DISCUSSION AND CONCLUSION

As in paper I, the models of Poveda & Woltjer (1968) and Duric & Seaquist (1986, hereafter D&S), after the convolution procedure, give $\Sigma - D$ relations which fail to closely match the empirical relations. The relations derived from theoretical considerations tend to have very flat slopes; e.g. the convolution of relation (7) with the D&S relation ($\Sigma \propto D^{-3.5}$) gives a $\Sigma - D$ relation of the form $\Sigma \propto D^{-1}$. Analogously, the relation obtained by Poveda & Woltjer ($\Sigma \propto D^{-3}$), when convolved with relation (7), yields a $\Sigma - D$ relation of the form $\Sigma \propto D^{-0.5}$. These two models modify the original Shkolvsky theory, while instead a modification of the Kesteven model gives the closest agreement with empirical data. We emphasize that the inconsistency between "flatter master" empirical relation (Urošević 2002, 2003) with $\beta \approx 2$ and the "flatter" theoretical relation of D&S with $\beta = 3.5$ is approximately the same as the inconsistency between the same relation and the relation for the modified model of D&S (a difference of $\beta \approx 1$ in both cases). Therefore, as in Paper I, the latest theoretical relation is of the same level of inconsistency with the empirical one while the initial relations (Shklovsky 1960, Kesteven 1968) are significantly improved.

As in Paper I, the modified Shklovsky theory gives a relation which is closer to the empirical relations. Kesteven's modified theoretical relation gives the best agreement with the updated "master" empirical $\Sigma - D$ relation (Urošević 2002, 2003).

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МОДИФИКОВАНА ТЕОРИЈСКА $\Sigma - D$ РЕЛАЦИЈА: II. СЛУЧАЈ ПРОМЕНЉИВЕ ТЕМПЕРАТУРЕ УНУТАР ОСТАТКА СУПЕРНОВЕ

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Претходно саопштење

Модификација теоријске $\Sigma - D$ релације за остатке супернових звезда у адијабатској фази ширења заснована је на конволуцији првобитне релације, коју је извео Шкловски, са новим обликом $\Sigma - D$ релације за закочно зрачење јонизованог гасног облака изведене у овом чланку. Као и у Чланку I, ми смо користили модел Мекија и Острајкера за остатке супернових и међузвездану средину. Кестеновa модификована теоријска релација даје најбоље слагање са најновијом "укупном" емпиријском $\Sigma - D$ релацијом коју је извео Урошевић.