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ON THE APPLICATION OF A PARTICULAR FORM OF THE GRAVITATIONAL POTENTIAL

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SUMMARY: A formula, already existing in the literature, describing the gravitational potential of a spherically symmetric stellar system is analysed. Some interesting cases of comparison with empirical formulae are presented. A good agreement is found.

Key words. Galaxy: kinematics and dynamics – methods: analytical

1. INTRODUCTION

In some earlier studies of the present author (e. g. Ninković 2001a; Ninković 2001b) the importance of the results by Tartu astronomers (Kuzmin and Malasidze 1970; Kuzmin and Veltmann 1973) was indicated. In the present paper one of their formulae for the gravitational potential of a spherically symmetric stellar system was treated in an unconventional way.

2. THE CASE UNDER STUDY

The formula referred to in Introduction is the following

$$\Pi = \frac{G\kappa M}{a + [(\kappa r)^2 + r_c^2]^{1/2}} \ . \tag{1}$$

Here Π is the potential, r – the distance to the centre, G – the universal gravitation constant, the other quantities (constants) being the parameters: κ is dimensionless ($\kappa > 0$), M has dimension of mass, r_c is the scale length of the system, and a may be considered as the auxiliary scale length.

If $a \geq 0$, then the volume of the system can be infinite and M will be its total mass. If a were negative, but $|a| < r_c$, then the system density would vanish at a finite distance to the centre, to become negative farther on. This was, of course, noticed by the proposers. However, the case can be of interest since the system need not cover an infinite volume. It is well known that there are models describing mass distribution within a stellar system where the density vanishes at a finite distance to the centre. The best known among them is, certainly, the King (e. g. 1962) model applied very often to star clusters and dwarf galaxies. Therefore, with regard to the relatively simple form of potential (1) any further suitable modification of this formula could be useful for the purpose of describing the mass distribution in real stellar systems. It is clear that in such a case the potential for distances over the limiting radius (where density vanishes) cannot be described by Eq. (1); instead for these distances one should use the well-known relation for the point-mass potential.

3. RESULTS

It should be noted that formula (1) contains two parameters of importance: κ and a. The scale length r_c, when (1), compared to another mass distribution, is assumed as the distance unit and as equal to the scale of this other distribution. The present procedure consists of two parts - cumulative-mass calculation and density calculation. When a is negative, then the critical distance of density vanishing can be established through the cumulative mass, that is reading the distance where it the maximum. Therefore, the parameter M need not coincide with the total mass of the system. The ratio of these two masses depends on the ratio of limiting radius to scale length.

For a chosen a one solution is not enought for this critical distance as it depends also on κ. As an additional comment it may be said that, perhaps, a solution where at the limiting radius the density is still non-zero, would seem more physical, but the density-vanishing distance is surely an upper limit beyond which the system can no longer exist.

In order to make this presentation more clear some dimensionless parameters are introduced. First, a is expressed in terms of r_c, $\alpha = a/r_c$ ($-1 < \alpha < 0$), whereas the additional dimensionless parameters are λ and x_l - $\lambda = \mathcal{M}/M$ ($\mathcal{M}$ total mass), $x_l = r_l/r_c$ (r_l limiting radius). It is clear that λ and x_l are not independent parameters, they can be obtained whenever α and κ are known. It should be noted that λ does not depend on κ. This can be seen from the following expression for x_l

$$x_l^2 = \frac{(\frac{-3\pm\sqrt{9-8\alpha^2}}{4\alpha})^2 - 1}{\kappa^2} . \qquad (2)$$

This formula derives from the condition of the cumulative-mass maximum. As for the cumulative-mass formula, it is, as well known, obtained by differentiating the formula for the potential (1). Since the square of the dimensionless radius x enters the formula for the cumulative mass, just as (1), multiplied by κ^2, the statement that λ is independent of κ becomes obvious. It is also to be noted that x_l is inversely proportional to κ, thus for the same α one will have for lower κ, a higher x_l.

It is understandable that also the Poisson equation can be solved for the present case, i. e. one can obtain the density

$$\rho(x) = \frac{M\kappa^3}{4\pi r_c^3} \frac{1}{(\alpha + \sqrt{(\kappa x)^2 + 1})^2 \sqrt{(\kappa x)^2 + 1}} [3 -$$

$$\frac{(\kappa x)^2}{\sqrt{(\kappa x)^2 + 1}} (\frac{2}{\alpha + \sqrt{(\kappa x)^2 + 1}} + \frac{1}{\sqrt{(\kappa x)^2 + 1}})] . \qquad (3)$$

It should be noted that this formula is valid for $x \leq x_l$ only, beyond that the density is, of course, equal to zero. By substituting here $x = 0$ one finds the relation between M and the product $\rho(0) r_c^3$.

With the upper limit for the system outer radius is established in the described way one can undertake comparison to an empirically based mass distribution. As such for the present analysis is chosen the mass distribution proposed by King (1962). As already said above, it has been very often applied, especially for the purpose of fitting the observed spatial distribution in star clusters and dwarf galaxies. However, for the case of the King mass distribution there are no analytical solutions, either for the cumulative mass or for the potential. This fact, certainly, renders such a comparison even more interesting.

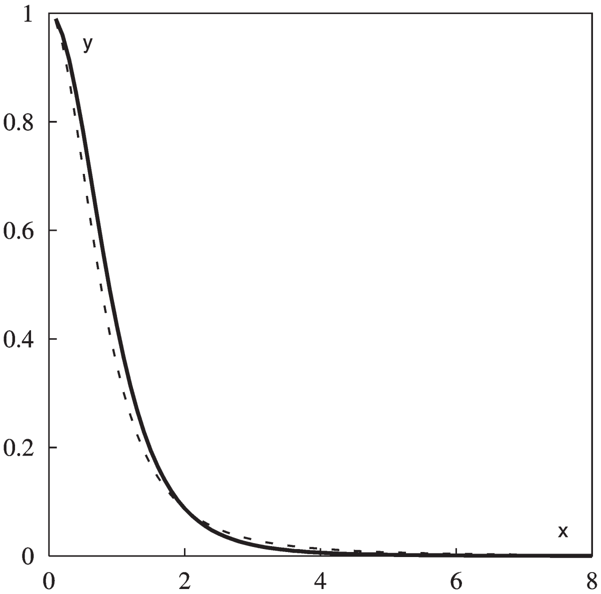

Fig. 1. *Density dependence: y is dimensionless density - $y = \rho/\rho(0)$, x is dimensionless radius - $x = r/r_c$, the solid curve corresponds to density expressed by Eq. (3) with parameters $\alpha = -0.04$, $\kappa = 0.63$, the dashed curve corresponds to the King mass distribution, $r_l/r_c = 60$ where the central densities and radii r_c are equal to each other.*

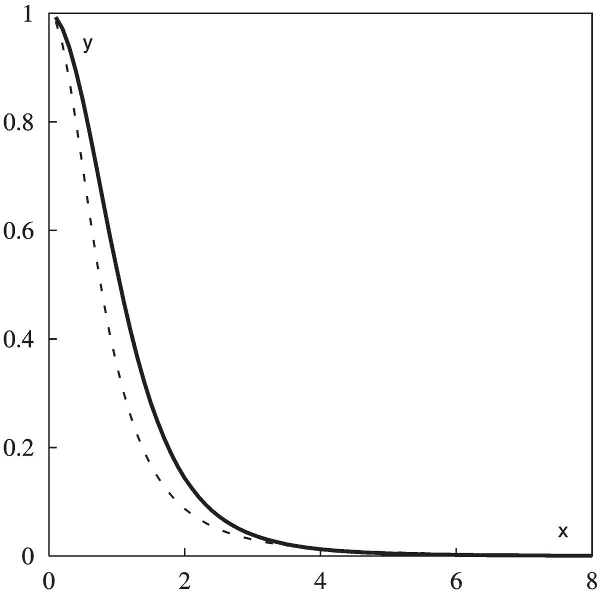

Fig. 2. *The same as Fig. 1. but with parameters $\alpha = -0.048$, $\kappa = 0.53$. Though the agreement may seem worse than in the former case, the resulting total masses are almost identical.*

The first step in comparison is to calculate the total mass for both distributions, expressed in terms of the central density and scale (r_c). These two quantities are assumed equal for both distributions. In the comparison with the King's mass distribution the equality is required for the total masses (the same $\rho(0)$ and r_c) and for the ratio r_l/r_c. Then the densities are calculated where the central density is used as unit. The fits are presented in Figs. 1 and 2.

4. DISCUSSION AND CONCLUSIONS

The present paper is aimed at finding a formula describing the gravitational potential suitable for use, especially taking into account the calculation of star orbits. The mass distribution corresponding to formula (1) is compared to the mass distribution proposed by King (1962). The reason of insisting on the comparison to this formula is that King's distribution is widely used, especially in studies of star clusters and dwarf galaxies. In addition, this distribution has an empirical base (the surface-density formula - King 1962) and later on it has been more generally treated by the proposer in terms of the phase space (e. g. King 1966). On the other hand, its serious shortcoming is, no doubt, the impossibility to obtain the cumulative mass and the potential analytically. In this matter there have been some efforts, for instance the connection between the King distribution and the generalised Schuster density law (the concrete case $i = 3$, for details see Ninković 1998). It should certainly, be pointed out that this density law results in very simple formulae, especially for the deriving of corresponding expressions for the surface density which certainly is an important advantage.

The similarity between these two mass distributions was noticed as early as by Veltmann (1961). To be more precise, the space densities expressed in terms of the central density are almost identical in the inner parts provided that the particular case of the King distribution pertains a high ratio of the limiting radius (called "tidal" by King) to the scale length ("core radius" in King's articles). However, in the periphery appear essential differences. The nature of these differences becomes clear if emphasized that in King's case at the limiting radius the density vanishes, whereas in the other one the density at the limiting radius is different from zero. Briefly told, as written in Ninković (1998), King's density formula appears as an "asymptotical case" of the given generalised Schuster density law.

With regard to the fact that in King's case there are no analytical solutions for the potential, i. e. cumulative mass, another attempt (Živkov and Ninković 1998) has been made where formulae were used similar to those applied in the particular case of the generalised Schuster density law, but it resulted in some rather cumbersome expressions for the gravitational potential compared to formula (1).

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REFERENCES

King, I.: 1962, *Astron. J.*, **67**, 471.
King, I.R. 1966, *Astron. J.*, **71**, 64.
Kuzmin, G.G. and Malasidze, G.A.: 1970, *Publ. Tart. Astrof. Obs. im. V. Struve*, **38**, 181.
Kuzmin, G.G. and Veltmann, Ü.-K.: 1973, *Publ. Tart. Astrof. Obs. im. V. Struve*, **40**, 281.
Ninković, S.: 1998, *Serb. Astron. J.*, **158**, 15.
Ninković, S.: 2001a, *Serb. Astron. J.*, **163**, 1.
Ninković, S.: 2001b, *Serb. Astron. J.*, **164**, 17.
Veltmann, Ü.-K.: 1961, *Publ. Tart. Astron. Obs.*, **33**, 387.
Živkov, V. and Ninković, S.: 1998, *Serb. Astron. J.*, **157**, 7.

О ПРИМЕНИ ЈЕДНОГ КОНКРЕТНОГ ГРАВИТАЦИОНОГ ПОТЕНЦИЈАЛА

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Оригинални научни рад

Анализира се једна формула, која већ постоји у литератури, а служи за описивање гравитационог потенцијала у сферно симетричном звезданом систему. Представљени су неки интересантни случајеви поређења са емпиријским формулама. Констатовано је добро слагање.