Serb. Astron. J. № 165 (2002), 9 – 16

UDC 521.17 : 52-59
Original scientific paper

BINARY COLLISIONS IN POPOVICI'S PHOTOGRAVITATIONAL MODEL

V. Mioc[1] and C. Blaga[2]

[1] *Astronomical Institute of the Romanian Academy*
Str. Cuţitul de Argint 5, RO-75212 Bucharest, Romania

[2] *Astronomical Institute of the Romanian Academy*
Astronomical Observatory Cluj-Napoca
Str. Cireşilor 19, RO-3400 Cluj-Napoca, Romania

(Received: February 15, 2002)

SUMMARY: The dynamics of bodies under the combined action of the gravitational attraction and the radiative repelling force has large and deep implications in astronomy. In the 1920s, the Romanian astronomer Constantin Popovici proposed a modified photogravitational law (considered by other scientists, too). This paper deals with the collisions of the two-body problem associated with Popovici's model. Resorting to McGehee-type transformations of the second kind, we obtain regular equations of motion and define the collision manifold. The flow on this boundary manifold is wholly described. This allows to point out some important qualitative features of the collisional motion: existence of the black-hole effect, gradientlikeness of the flow on the collision manifold, regularizability of collisions under certain conditions. Some questions, coming from the comparison of Levi-Civita's regularizing transformations and McGehee's ones, are formulated.

1. INTRODUCTION

The dynamics of the components of the solar system is mainly ruled by gravitation.However, nongravitational forces also influence the motion. Among these, the direct solar radiation pressure plays a central role. If for bodies with small area-to-mass ratio (planets, satellites, asteroids) the repelling radiative force is negligible as compared to the gravitation, there equally are bodies (particles in cometary tails, dust grains) for which the radiation influence become considerable and can even prevail over the gravitation.

The interest in dynamical effects of the radiation pressure goes as far back as the early 1600s; Kepler himself, in his *Harmonices mundi* (1619), studied this problem. Later on, many outstanding scientists focused on this topic; it is sufficient to mention among them Newton, Huygens, Coulomb, or Maxwell.

An interesting practical problem in this context was addressed by Tsiolkovsky in 1921 and Tsander in 1924. Based on Lebedev's experiments (1899 – 1907), which allowed to measure the radiative force, they imagined interplanetary spacecrafts, endowed with large mirrors, driven by the direct solar radiation pressure. Hermann Oberth also dealt with this problem in 1923, resuming it three decades later, in 1954. Since then, the subject was largely discussed (see Anisiu 1995 and the references therein).

In the first quarter of the 20th century, the Romanian astronomer Constantin Popovici proposed a modification of the inverse-square law that results from the combination of the Newtonian attraction and the repelling force of radiation. He added a term depending on the radial velocity and on the speed of light (Popovici 1923), which made his law – as he wrote – applicable to "radial attractive or repulsive (whether gravitational, electrical, or light repulsion)" forces. The physical background of this modification is the change in the energy of photons due to the change of wavelength, due in turn to distance variation. Later on, Popovici (1940) resumed this model, describing explicitly the motion of a body in the photogravitational field of a star.

Popovici's model was independently used by Armellini (1937) (but within a much less realistic framework, involving only the gravitational force) and Chazy (1939). Moreover, Nadolschi and Plăcinţeanu (1940) applied this law to the motion of electrons around the atomic nucleus.

In this paper we start a systematic study of the two-body problem in Popovici's photogravitational field. Since the model presents singularities corresponding to collisions, we shall deal here with the collisional motion.

Section 2 presents Popovici's model in the form considered by Anisiu (1995). The equations of motion of the two-body problem associated with this model, as well as the first integral of the angular momentum, are established in Section 3.

To remove the collision singularity, in Section 4 we resort to a set of McGehee-type transformations of the second kind (McGehee 1974). Under these transformations, the equations of motion become regular. Moreover, the singularity is blown up and replaced by the so-called *collision manifold*, pasted on the phase space instead of it.

Section 5 describes the flow on the collision manifold. This flow is deprived of physical significance, but – due to the continuity of solutions with respect to initial data – it offers information about the trajectories that neighbour collision. Two situations are considered: $C = 0$ and $C \neq 0$ (where C stands for the angular momentum constant). In the first case (radial motion), the flow on the collision manifold (homeomorphic to a cylinder) consists of two circles of degenerate equilibria (UC = the upper circle, and LC = the lower circle), heteroclinic orbits that move from UC to LC, and trajectories that leave UC or tend to LC. For nonzero angular momentum, the flow on the collision manifold has different structures, according to the increase of $|C|$. First, the phase portrait imitates the one corresponding to $C = 0$, with the difference that the circles of degenerate equilibria are replaced by periodic orbits. As C increases in module, the two periodic orbits approach each other, and coincide for a critical value of $|C|$ (entailing the existence of a single periodic orbit and the vanishing of the heteroclinic trajectories). Lastly, when $|C|$ exceeds the critical value, the periodic orbit vanishes, and the flow consists only of trajectories that wind upwards around the cylinder.

Section 6 formulates some conclusions, emphasizing the main qualitative features of the collisional motion: the existence of the *black-hole effect* (spiral collision), the gradientlikeness of the flow on the collision manifold, the nonregularizability of collisions for $|C|$ smaller than or equal to the critical value.

Section 7 addresses a challenge. Regularizing the motion equations and the angular momentum integral via Levi-Civita's transformations, we are led to a comparison between these and McGehee's blow up. Is there a relationship between these transformations? If yes, what is this relationship? The answer to these questions is still unknown.

One could ask: why make Popovici's model revive, even if resumed by illustrious astronomers as Armellini and Chazy? There are several reasons for it. On the one hand, it is a physically realistic model, which adds a well-known effect (even if very small) to the classical model, without trying to replace it. On the other hand, Popovici's model joins two-body problems with drag or thrust that were tackled by a lot of authors: Poynting (1903), Plummer (1905, 1906), Robertson (1937), Wyatt and Whipple (1950), Brouwer and Hori (1961), Mittleman and Jezewski (1982), Mavraganis (1991), Mavraganis and Michalakis (1994); for more details, see Burns *et al.* (1979), Mignard (1992), Diacu (1999). Lastly, our paper points out the usefulness of the powerful tool of McGehee transformations in describing the behaviour of the orbits that approach collision for no matter what kind of field.

2. POPOVICI'S MODEL

Let us write the Newtonian attractive force (in module) in the generic form

$$|\mathbf{F}_N| = -A/r^2, \tag{1}$$

with $A > 0$. Also, let us consider for the radiative repelling force (in module, too) the generic expression

$$|\mathbf{F}_R| = R/r^2, \tag{2}$$

with $R > 0$. The photogravitational resulting force (of mixed nature) obviously is

$$|\mathbf{F}_{pg}| = -k/r^2, \tag{3}$$

where k stands for $A - R$.

Of course, the photogravitational force can be attractive or repulsive, as k is positive or negative, respectively. This depends on the characteristics of both field-generating body (especially luminosity) and "satellite" body (especially area-to-mass ratio). In the solar system, for instance, k is positive for the interaction Sun-planets, but it can be negative in the case of micrometeoric dust, particles in cometary tails, or solar sails.

There were lots of papers that treated the two-body problem associated with a force of the type (3) (for a survey, see, e.g., Polyakhova 1986; McInnes 1991). Moreover, the much more general case of $k = k(t)$ or $k = k(\theta)$, where t is the time and θ denotes a polar angle, was also tackled by many authors, among which we arbitrarily quote: Gyldén (1884), Messchersky (1902), Jeans (1924), Savedoff and Vila (1964), Saari (1977), Saslaw (1978), Şelaru *et al.* (1992, 1993), etc.

Let us come back to the law (3) with $k =$ constant. Popovici's contribution consists of the new form given to this law

$$|\mathbf{F}_{pg}| = -\frac{A}{r^2} + \frac{R}{r^2} - \frac{R\dot{r}}{cr^2}, \tag{4}$$

where $\dot{r}$ denotes the velocity vector projection on the radius vector, while c is the speed of light.

What is the physical background of Popovici's model? Let us consider a pointlike source that emits a radiation of intensity I and wavelength λ. Let $\Delta\sigma$ be a surface element that receives normally the energy ΔQ. Then, if we take into account the fact that a change in r leads to a change in λ, hence to a change in the energy of photons that fall on $\Delta\sigma$, the radiative energy received by $\Delta\sigma$ in the time unit will approximately be $\Delta Q = I(1 - \dot{r}/c)\Delta\sigma/(4\pi r^2)$. This leads straightforwardly to the last two terms in the right-hand side of formula (4).

Armellini (1937) proposed a similar law, but much less realistic from the physical standpoint, involving the gravitational force only. He assigned to (1) the modified form $|\mathbf{F}| = -A(1 + \varepsilon\dot{r})/r^2$ (ε being an extremely small positive constant), claiming that "a large number of cosmogonical problems can be immediately explained in this way". However, Armellini emphasized that his "law" is a simple hypothesis.

A more condensed form given by Popovici to his law is

$$|\mathbf{F}_{pg}| = -\frac{k(1 + \varepsilon\dot{r})}{r^2}, \tag{5}$$

in which $\varepsilon = R/(ck)$ and $k \neq 0$. The shortcoming of (5) is that, for $k = 0$, (4) and (5) are no more equivalent.

Pointing out this shortcoming, Anisiu (1995) proposed a unitary form for (4) and (5), namely

$$|\mathbf{F}_{pg}| = -\frac{k + q\dot{r}}{r^2}, \tag{6}$$

where $q = R/c > 0$. In the case $k = 0$, the expression (5) is obtained from (6) by putting $\varepsilon = q/k$.

3. EQUATIONS OF MOTION AND THE FIRST INTEGRAL

The two-body problem associated with a force of the type (6) can be reduced to a central-force problem. Fixing the field-generating source at the origin of a polar coordinate system, the equations that describe the relative motion of the "satellite" body read

$$\ddot{r} - r\dot{\theta}^2 = -\frac{k + q\dot{r}}{r^2}, \tag{7}$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0. \tag{8}$$

where the dot marks differentiation with respect to the time t. It is clear that the motion is confined to a plane.

Since the force field is a central one, the equations of motion admit the first integral of angular momentum

$$r^2\dot{\theta} = C, \tag{9}$$

immediately obtainable from (8). In (9), C stands for the angular momentum constant.

The most important consequence of the existence of the term in $\dot{r}$ in the expression of the force is the non-conservation of the total energy. Consequently, a first integral of energy does not exist (as in the classical photogravitational case).

Equations (7) and (9) constitute the basis for our qualitative endeavours. In the sequel, the field-generating source and the "satellite" body will be called *centre* and *particle*, respectively.

4. McGEHEE-TYPE TRANSFORMATIONS

The equations of motion present an isolated singularity at $r = 0$. Using a Painlevé-type criterion, one can prove that this singularity corresponds to a collision particle-centre (Mioc and Stavinschi 2000b, 2002).

To remove this singularity, we shall apply McGehee-type transformations of the second kind (McGehee 1974). For the first step, we consider the real analytic diffeomorphisms

$$\begin{aligned} r\dot{r} &= u, \\ r^2\dot{\theta} &= v. \end{aligned} \tag{10}$$

Under these transformations, which scale down the polar components of the velocity, the equations of motion become

$$\begin{aligned} \dot{r} &= \frac{u}{r}, \\ \dot{\theta} &= \frac{v}{r^2}, \\ \dot{u} &= \frac{u^2 + v^2 - qu}{r^2} - \frac{k}{r}, \\ \dot{v} &= 0, \end{aligned} \tag{11}$$

whereas the first integral of angular momentum acquires the form

$$v = C. \tag{12}$$

The singularity at $r = 0$ still persists, therefore the second step of the McGehee-type transformations consists of the real analytic diffeomorphism

$$dt = r^2 ds, \tag{13}$$

which rescales the time. Keeping, by abuse, the same notation for the new functions of the timelike variable s, the motion equations become

$$\begin{aligned} r' &= ru, \\ \theta' &= v, \\ u' &= u^2 + v^2 - qu - kr, \\ v' &= 0, \end{aligned} \tag{14}$$

where $' = d/ds$. Of course, the angular momentum integral preserves its form (12).

5. COLLISION MANIFOLD

One observes that the equations of motion (14) are now regular. This means that the phase space can be analytically extended to contain the boundary manifold

$$M_{col} = \{(r, \theta, u, v) \mid r = 0\}. \tag{15}$$

Let us consider C as a parameter. We can therefore define the constant-angular-momentum manifold

$$M_C = \{(r, \theta, u, v) \mid v = C\}. \tag{16}$$

The so-called *collision manifold* is defined as the intersection $M_0 = M_{col} \cap M_C$, namely

$$M_0 = \{(r, \theta, u, v) \mid r = 0, \quad \theta \in S^1, \quad u \in \mathbb{R}, \quad v = C\}. \tag{17}$$

In this way, we have blown up the singularity at $r = 0$ and pasted the manifold M_0, instead of it, on the phase space.

An important remark is to be made here: collisions are possible not only for $C = 0$ (as in the classical Newtonian model), but also for nonzero C. Collisions for $C = 0$ are rectilinear; in case $C \neq 0$, they are spiral (the so-called *black-hole effect*).

By the manner in which we have defined the collision manifold, for every fixed value of the angular momentum constant, M_0 is homeomorphic to a strip of width 2π in the (θ, u)-plane. Since $\theta \in S^1$ (the segment $[0, 2\pi]$ with the end points pasted together), the strip M_0 can also be identified with a 2D cylinder. Both the strip and the cylinder actually are imbedded in the 4D full phase space of the coordinates (r, θ, u, v).

The remainder of this paper will deal with the description of the flow on the collision manifold. This flow is deprived of physical significance, but – due to the continuity of solutions with respect to the initial conditions – it provides valuable information about the orbits that approach collision.

By (14) and (17), the vector field on M_0 reads

$$\begin{aligned} \theta' &= C, \\ u' &= u^2 - qu + C^2. \end{aligned} \tag{18}$$

Consider the radial motion case ($C = 0$). In this case, $\theta = \theta_0$, with arbitrary $\theta_0 \in S^1$, and the flow on the M_0 cylinder consists of:

- two circles formed by degenerate equilibria: the lower circle LC ($\theta = \theta_0$, $u = 0$) and the upper circle UC ($\theta = \theta_0$, $u = q$);
- heteroclinic orbits that move downwards, from UC to LC;
- orbits that move upwards, tending to LC or ejecting from UC.

The phase portrait of this case is sketched in Fig. 1.

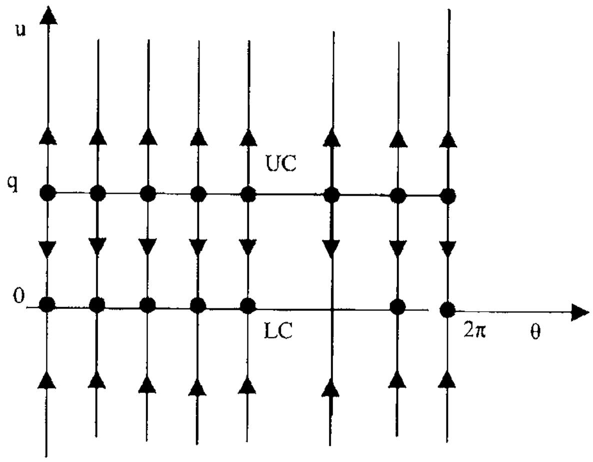

Fig. 1. *The flow on the M_0 strip for $C = 0$.*

Consider now the nonrectilinear motion ($C \neq 0$). In the case $0, |C| < q/2$, equations (18) easily lead to the dependence

$$u(\theta) = \frac{q}{2} - \frac{\sqrt{q^2 - 4C^2}}{2} \tanh\left(\frac{\sqrt{q^2 - 4C^2}}{2C}\theta + K_1\right), \tag{19}$$

where the integration constant K_1 is determined from the initial conditions $\theta(s = s_0)$, $u(s = s_0)$.

The flow on the M_0 cylinder consists of:

- two periodic orbits (relative equilibria): the lower one (LPO) at $u_1 = (q - \sqrt{q^2 - 4C^2})/2$, and the upper one (UPO) at $u_2 = (q + \sqrt{q^2 - 4C^2})/2$ (it is clear that $0 < u_1 < u_2 < q$);
- heteroclinic orbits that move downwards, ejecting asymptotically fromUPO and tending asymptotically to LPO;
- orbits that move upwards, tending asymptotically to LPO or ejecting asymptotically from UPO.

The phase portrait of this case is plotted in Fig. 2.

Consider the special value $|C| = q/2$. Equations (18) lead to

$$u(\theta) = \frac{q}{2}\left(1 - \frac{\delta}{\theta + K_2}\right), \tag{20}$$

where K_2 is an integration constant, determinable from the initial data, whereas

$$\delta = \begin{cases} 0, & \text{for } u = q/2; \\ 1, & \text{for } u \neq q/2. \end{cases} \tag{21}$$

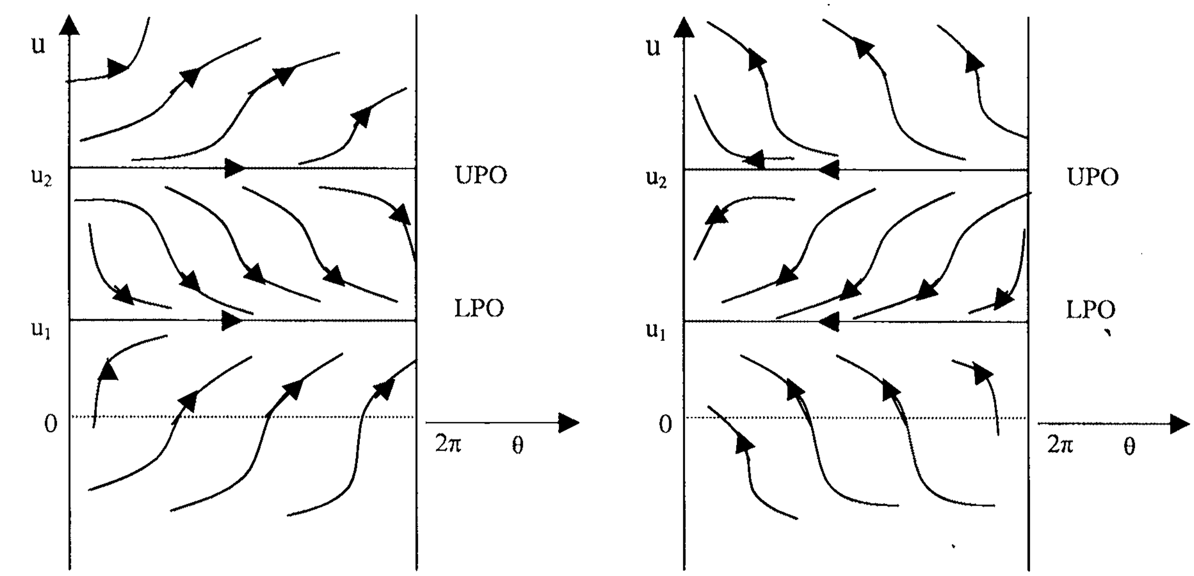

Fig. 2. *The flow on the* M_0 *strip for* $0 < C < q/2$ *(left) and* $-q/2 < C < 0$ *(right).*

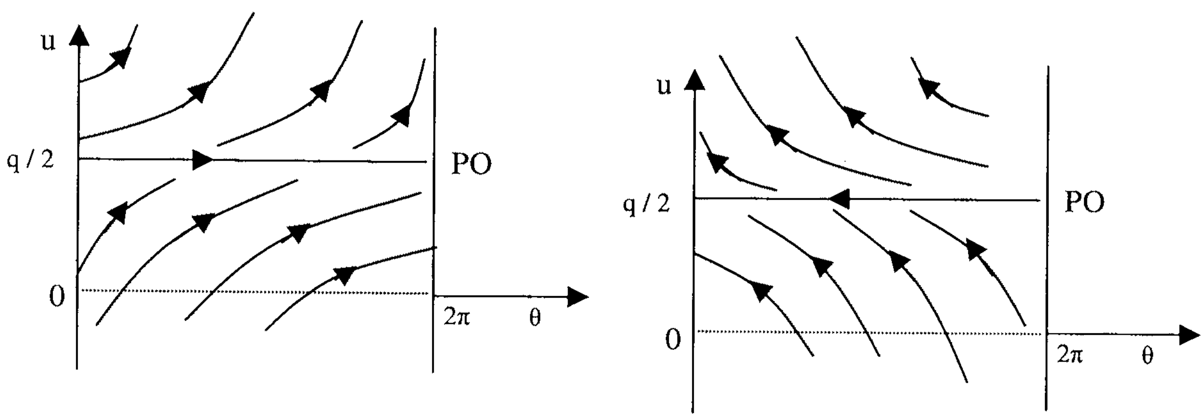

Fig. 3. *The flow on the* M_0 *strip for* $C = q/2$ *(left) and* $C = -q/2$ *(right).*

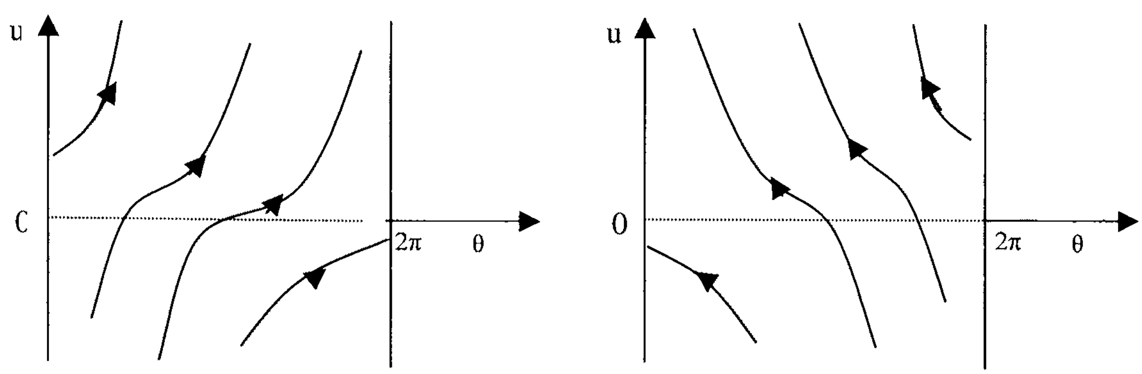

Fig. 4. *The flow on the* M_0 *strip for* $C > q/2$ *(left) and* $C < -q/2$ *(right).*

The flow on the M_0 cylinder consists of:

- one periodic orbit (relative equilibrium): PO at $u = q/2$;
- orbits that wind upwards around the cylinder, tending asymptotically to PO, or ejecting asymptotically from it.

The phase portrait in this case is represented in Fig. 3.

Lastly, let us consider the situation $|C| > q/2$. Equations (18) lead to

$$u(\theta) = \frac{q}{2} + \frac{\sqrt{4C^2 - q^2}}{2} \tan\left(\frac{\sqrt{4C^2 - q^2}}{2C}\theta + K_3\right), \tag{22}$$

where the integration constant K_3 is determined from the initial conditions.

The flow on the M_0 manifold now consists only of orbits that wind upwards around the cylinder; they look like in Fig. 4.

6. CONCLUDING REMARKS

The above results allow to formulate some concluding remarks:

- Collisions in the two-body problem associated with Popovici's field occur not only for zero angular momentum (radial motion), but also for nonzero angular momentum (spiral motion with black-hole effect). This was already pointed out for some classical potentials (e.g., Wintner 1941; McGehee 1981; Diacu *et al.* 1995; Stoica and Mioc 1997; Mioc and Stavinschi 2000a), but not for a field of such a structure yet.

- For $C = 0$, the two circles of degenerate equilibria on M_0 are situated at the maximum reciprocal distance (q, on the u-axis). For $0 < |C| < q/2$, they become relative equilibria (periodic orbits) and approach each other as $|C|$ increases. For $|C| = q/2$, the two periodic orbits coincide. Finally, for $|C| > q/2$, they disappear.

- For $|C| \geq q/2$, the flow on the collision manifold is gradientlike with respect to the u-coordinate.

- Having in view the motion equations (14) for the full phase space, it is clear that collisions ($r \to 0$ in the future) may occur only for $u < 0$, whereas ejections ($r \to 0$ in the past) may occur only for $u > 0$. This means that a collision orbit may reach M_0 only in the region below the axis $u = 0$, while an ejection orbit may leave M_0 only in the region above this axis. This further means that collision solutions are not regularizable for $|C| \leq q/2$ (as in other cases, illustrated, e.g., by Diacu *et al.* 2000, or Mioc and Stavinschi 2001), but become regularizable for $|C| > q/2$.

7. APPENDIX: A CHALLENGE

In this study, to avoid the singularities, we resorted to the McGehee-type transformations. But we could use a lot of regularizing transformations. In this section we shall apply Levi-Civita's (1903) transformations (see also Aarseth and Zare 1974; Zare 1974; Şelaru 1997a,b) − based on Euler's (1767) regularization − to the motion equations, just to compare the results and to address some questions.

Let us apply Levi-Civita's transformations to the motion equations (7)−(8). These transformations are

$$\begin{aligned} r &= \xi^2, \\ \dot{r} &= \frac{\eta}{\xi}, \\ \dot{\theta} &= y, \end{aligned} \tag{23}$$

and make the motion equations read

$$\begin{aligned} \dot{\xi} &= \frac{\eta}{2\xi^2}, \\ \dot{\eta} &= \frac{\eta^2}{2\xi^2} + \xi^3 y^2 - \frac{k}{\xi^3} - \frac{q\eta}{\xi^4}, \\ \dot{\theta} &= y, \\ \dot{y} &= -\frac{2\eta y}{\xi^3}, \end{aligned} \tag{24}$$

while the angular momentum first integral becomes

$$\xi^4 y = C. \tag{25}$$

Since the singularity at $\xi = 0$ persists, we proceed to a reparameterization of the time via

$$dt = \xi^4 d\tau, \tag{26}$$

which leads to the following equations of motion:

$$\begin{aligned} \frac{d\xi}{d\tau} &= \frac{1}{2}\xi^2\eta, \\ \frac{d\eta}{d\tau} &= \frac{1}{2}\xi\eta^2 + \xi^7 y^2 - k\xi - q\eta, \\ \frac{d\theta}{d\tau} &= \xi^4 y, \\ \frac{dy}{d\tau} &= -2\xi\eta y, \end{aligned} \tag{27}$$

where we preserved, by abuse, the same notation for the new functions of τ.

Having in view the first integral of angular momentum, which keeps its form, equations (27) can also read

$$\begin{aligned} \frac{d\xi}{d\tau} &= \frac{1}{2}\xi^2\eta, \\ \frac{d\eta}{d\tau} &= \frac{1}{2}\xi\eta^2 + C\xi^3 y - k\xi - q\eta, \\ \frac{d\theta}{d\tau} &= C, \\ \frac{dy}{d\tau} &= -2\xi\eta y. \end{aligned} \tag{28}$$

Comparing these equations with the motion equations (14), two questions arise:

- Is there a relationship between Levi-Civita's transformations and McGehee's blow up?
- If yes, what is this relationship?

These questions represent a challenge for the research we have performed in this paper. The answer will be given elsewhere.

Acknowledgements – The authors are indebted to Professor Ernesto Pérez-Chavela for carefully reading the manuscript and for the useful suggestions as regards Levi-Civita's regularizing transformations. Thanks are also due to the anonymous referee for help in improving the manuscript.

REFERENCES

Aarseth, S.J., Zare, K.: 1974, *Celest. Mech.* **10**, 185.
Anisiu, M.-C.: 1995, *Rom. Astron. J.* **5**, 49.
Armellini, G.: 1937, *Rendiconti Accad. Naz. Lincei* **26**, 209.
Brouwer, D., Hori, G.: 1961, *Astron. J.* **66**, 193.
Burns, J.A., Lamy, P.L., Soter, S.: 1979, *Icarus* **40**, 1.
Chazy, J.: 1939, *C. R. Acad. Sci. Paris* **209**, 133.
Diacu, F.: 1999, *Celest. Mech. Dyn. Astron.* **75**, 1.
Diacu, F.N., Mingarelli, A., Mioc, V., Stoica, C.: 1995, in R. P. Agarwal (ed.), *Dynamical Systems and Applications*, World Scientific Series in Applicable Analysis, Vol. 4, World Scientific, Singapore, 213.
Diacu, F., Mioc, V., Stoica, C.: 2000, *Nonlinear Analysis* **41**, 1029.
Euler, L.: 1767, *Novi Comm. Acad. Sci. Imp. Petropolis* **11**, 144.
Gyldén, H.: 1884, *Astron. Nachr.* **109**, No. 2593-94, col. 1.
Jeans, J.: 1924, *Mon. Not. Roy. Astron. Soc.* **85**, 2.
Levi-Civita, T.: 1903, *Ann. Math.* **9**, 1.
McGehee, R.: 1974, *Invent. Math.* **27**, 191.
McGehee, R.: 1981, *Comment. Math. Helvetici* **56**, 524.
McInnes, C.R.: 1991, *Vistas Astron.* **34**, 369.
Mavraganis, A.G.: 1991, *Celest. Mech. Dyn. Astron.* **51**, 395.
Mavraganis, A.G., Michalakis, D. G.: 1991, *Celest. Mech. Dyn. Astron.* **58**, 393.
Messchersky, J.: 1902, *Astron. Nachr.* **159**, No. 3807, col. 229.
Mignard, F.: 1992, in D. Benest, C. Froeschlé (eds), *Interrelations between Physics and Dynamics for Minor Bodies in the Solar System*, Ed. Frontières, Gif-sur-Yvette, 447.
Mioc, V., Stavinschi, M.: 2000a, *Rom. Astron. J.* **10**, 71.
Mioc, V., Stavinschi, M.: 2000b, *Rom. Astron. J.* **10**, 95.
Mioc, V., Stavinschi, M.: 2001, *Phys. Lett. A* **279**, 223.
Mioc, V., Stavinschi, M.: 2002, *Phys. Scripta* **65**, 193.
Mittleman, D., Jezewski, D.: 1982, *Celest. Mech.* **28**, 401.
Nadolschi, V., Plăcinţeanu, I.: 1940, *Ann. Sci. Univ. Jassy* **26**, 397.
Plummer, H.C.: 1905, *Mon. Not. Roy. Astron. Soc.* **65**, 229.
Plummer, H.C.: 1906, *Mon. Not. Roy. Astron. Soc.* **67**, 63.
Polyakhova, E.N.: 1986, in V. A. Egorov (ed.), *Space Flight Using a Solar Sail: the Problems and the Prospects*, ser. *Space Flight Mechanics*, Vol. 19, Nauka, Moscow (Russian).
Popovici, C.: 1923, *Bull. Astron., Mémoires et variétés*, **3**, 257.
Popovici, C.: 1940, *C. R. Acad. Sci. Paris* **210**, 39.
Poynting, J.H.: 1903, *Phil. Trans. Roy. Soc. A* **202**, 525.
Robertson, H.P.: 1937, *Mon. Not. Roy. Astron. Soc.* **97**, 423.
Saari, D.G.: 1977, *Celest. Mech.* **16**, 407.
Saslaw, W.: 1978, *Astrophys. J.* **226**, 240.
Savedoff, M.P., Vila, S.: 1964, *Astron. J.* **69**, 242.
Stoica, C., Mioc, V.: 1997, *Astrophys. Space Sci.* **249**, 161.
Şelaru, D.: 1997a, *Rom. Astron. J.* **7**, 27.
Şelaru, D.: 1997b, *Rom. Astron. J.* **7**, 169.
Şelaru, D., Cucu-Dumitrescu, C., Mioc, V.: 1992, *Astron. Nachr.* **313**, 257.
Şelaru, D., Cucu-Dumitrescu, C., Mioc, V.: 1993, *Astrophys. Space Sci.* **202**, 11.
Wintner, A.: 1941, *The Analytical Foundations of Celestial Mechanics*, Princeton University Press, Princeton, N. J.
Wyatt, S.P., Whipple, F.L.: 1950, *Astrophys. J.* **111**, 558.
Zare, K.: 1974, *Celest. Mech.* **10**, 207.

ДВОЈНИ СУДАРИ У ФОТОГРАВИТАЦИОНОМ МОДЕЛУ ПОПОВИЦИЈА

V. Mioc[1] и C. Blaga[2]

[1] *Astronomical Institute of the Romanian Academy*
Str. Cuţitul de Argint 5, RO-75212 Bucharest, Romania

[2] *Astronomical Institute of the Romanian Academy*
Astronomical Observatory Cluj-Napoca
Str. Cireşilor 19, RO-3400 Cluj-Napoca, Romania

УДК 521.17 : 52-59
Оригинални научни рад

Динамика тела изложених комбинованом дејству гравитационог привлачења и силе радијативног одбијања има велике и дубоке импликације у астрономији. Двадесетих година двадесетог века румунски астроном Константин Поповици је предложио модификован фотогравитациони закон (разматрали су га такође и други научници). Овај рад се бави сударима проблема двају тела у вези са Поповицијевим моделом. Прибегавајући трансформацијама МекГијевог типа друге врсте добијамо регуларне једначине кретања и дефинишемо скуп судара. Ток на овај гранични скуп је у потпуности описан. Ово дозвољава да се истакну неке важне квалитативне карактеристике сударног кретања: постојање ефекта црне рупе, ток на сударни скуп наликује градијенту, регуларизибилност судара под одређеним условима. Формулисана су нека питања која долазе од поређења регуларизујућих трансформација Леви-Цивита и МекГија.