

A VARIANCE-COMPONENTS ANALYSIS FOR THE LONGITUDE-NETWORK ADJUSTMENT

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SUMMARY: The problem of weights in the task of longitude-network adjustment has not been solved yet because the structure of the observational variances is unknown. During the adjustment of a part of the European Longitude Network in the framework of the project of including Belgrade in this network, new models of determining the components of observational variances were analysed. These models offer a better description of the variance structure so that the weights are closer to their true values than those used up to now.

1. INTRODUCTION

For the purpose of establishing a new astronomical longitude network covering the region of Europe (*European Longitude Network - ELN*) there were several campaigns involving precise measurements of longitude differences between the national reference stations in seven European countries: Germany, Italy, Spain, Holland, France, Portugal and Austria (Kaniuth and Wende 1980, 1983, Wende 1992). Between 1977 and 1980 there were two campaigns, whereas the last one including the stations of Munich, Vienna and Graz was performed in 1988. The observations were made by W. Wende, *DGFI Institute of the Bavarian Academy of Sciences*, with a Danjon astrolabe by using the method of equal zenith distances. Selected stars from the FK5 Catalogue were observed. (In connexion with this observational material see also: Perović and Cvetković, 1998).

The 1988 Campaign took place between **July 20** and **September 10** with a total of 23 observational nights. At each station all three groups of stars, 10, 11 and 12, within 52 measurement series, were

observed. The total number of observed *star transits is 1601*. The total number of **REG**istrations of the star **TRANS**its across the **F**ictive mean **W**ire (number of **RETRAFIW**-s) is $1601 \times 12 = 19212$. In the case of these observations the zenith distance was determined on the basis of registering the time of a star transit across the given almucantar. The precision of this time registering has been analysed by Perović and Cvetković (1998).

In order to form the project of including Belgrade in ELN the present authors use the results of the analysis of the measurements from the 1988 campaign. The first task posed and solved this time by them was the study of a mathematical model of adjusting the observations. In this paper they present a study of a stochastic model, more precisely the study of a model of weights. An equivalent task to this one is the study of a model of observational-variance components. Here the authors study four models, i. e. four models of variance components, as well as three functional adjustment models. The present study also includes Wende's model of observational weights.

Of course, the adjustment is achieved after introducing reductions in the observations. For this purpose the star positions from the Hipparcos Catalogue are used.

2. THE MATHEMATICAL MODEL

In the adjustment and analysing the accuracy of the measurements *the model of covariation analysis* is used:

$$\left. \begin{array}{l} (a) \text{ Linear : } v = Ax + Bt + f, \quad f = l_o + l \\ (b) \text{ Stochastic : } M[v] = 0, \\ M[vv^T] = K = \sigma^2 P^{-1} = \sigma^2 \text{diag}\{P_i^{-1}\}. \end{array} \right\} (1)$$

where: v – vector of measurement corrections; l – vector of measurements; l_o – vector of approximate values of measured quantities; x – vector of *basic parameters*; t – vector of *additional parameters*; A i B – matrices of known coefficients; σ^2 – variance coefficient, (in calculations assumed $\sigma^2 = 1$); K – variance-covariance matrix of measurements and P – matrix of measurement weights.

The relative observational weights are calculated as the quotient of the variance coefficient (root-mean-square error of unit weight) and the measurement variance:

$$P_l \equiv P_z = \frac{1}{\sigma_z^2} \quad (2)$$

where l is the general designation for a single observation; in this case it is the zenith distance z .

3. THE FUNCTIONAL MODELS

In the course of the study of the model of variance components the functional-model problem also arises so that there is a necessity to study this model, too.

For the purpose of studying an adequate functional regression model we use the functional model of covariance analysis, i.e. *equations of corrections* (1a):

$$v = Ax + Bt + f, \quad f = l_o - l \quad (3)$$

where we *study the influences of Bt* in the observations which can be described with the vector of additional parameters t , whereas the vector of basic parameters x is the same in all the functional models.

The vector of basic parameters x is the same in all of the three studied functional models and it has nine components: three latitude increments, three longitude increments (for stations Munich, Vienna and Graz) and three increments of zenith distance for the three observed star groups, 10, 11 and 12. Therefore, *the studied functional models differ with respect to the term Bt representing the effects of individual factors on the observations.*

The First Functional Model (FM1). It describes the effects of eight factors and, consequently, the vector t has 8 components: variation with time of latitude, variation with time of longitude, correction due to the change of the instrument temperature, correction due to the change of the difference between the instrument temperature and the outdoor one, correction due to the human-eye adaptation to the bright and dark, correction due to the effect of star colour and two corrections due to the effect of star magnitude m_v .

The Second Functional Model (FM2). Compared to Model FM1 this model is extended through taking into account the corrections due to the change of the instrument temperature and the ones due to the change of the difference between the instrument temperature and the outdoor one for every observing night (23 observing nights). Therefore, the t vector has 52 components.

The Third Functional Model (FM3). Compared to Model FM1 the t vector is extended with additional parameters which are the corrections to star right ascensions. The number of these corrections is 34.

4. THE STUDY OF THE MODEL OF VARIANCE COMPONENTS

The study of *the weight model* concerning the zenith-distance (z) measurements, in view of Eq. (2), is reduced to the studying of *the model of variance components* for the zenith-distance measurements and this will be the subject of our treatment.

Four models of measurement variance components (VC) are studied. They are:

1. Wende's Model of Variance Components (Wende, 1992) – (VCW);
2. Model with Two Components (Uralov, 1980) – (VC2);
3. "A" Model with Three Components – (VC3A),
and
4. "B" Model with Three Components – (VC3B);

Models 2 and 4 have not been applied earlier in the weight determination for the purpose of adjusting the longitude network. For this reason a comparison of the weights obtained on the basis of these models with those used in the earlier one is of interest to the present authors.

For the purpose of estimating the variance components *the MINQE method* (Rao, 1970; Perović, 1998) is used.

^{1°} **Wende's Model of Variance Components (VCW).** Wende assumed the same variance for all zenith-distance observations belonging to the same group. In this way in the adjustment of the observations he used three different variances only, one for each of the three groups (10, 11 and 12) of observed stars so that one obtains for the observational variance

$$\sigma_z^2 = \sigma_i^2, \quad i = 10, 11, 12. \quad (4)$$

Here the subscript i indicates observations within the same star group - 10, 11 or 12.

The MINQE estimates of the variance components σ_{10}^2 , σ_{11}^2 and σ_{12}^2 are given in Table 1.

Table 1. The MINQE VC estimates (σ_{10}^2 , σ_{11}^2 , σ_{12}^2) in Wende's model for three functional adjustment models, FM1, FM2 and FM3; (n – number of observations, u – number of parameters in a linear model).

Funct. Model	n	u	σ_{10}^2 [″ ²]	σ_{11}^2 [″ ²]	σ_{12}^2 [″ ²]
FM1	1378	17	0.028098	0.025609	0.026597
FM2	1396	61	0.019039	0.019626	0.020256
FM3	1377	95	0.015492	0.016709	0.016069

2° Model with Two Variance Components (VC2).

The random-error variance for the case of registering the time of star transit can be described by the following model (Uralov, 1980):

$$\sigma_t^2 = \sigma_2^2 + \frac{\sigma_1^2}{V^2}.$$

where σ_1^2 and σ_2^2 are the components of the observational variances, whereas V is the velocity of motion of the observed star ($V = \cos \varphi \sin A$).

Using also the cosine theorem $\cos z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos t$ and the relation $\sin A = \cos \delta \sin t / \sin z$ one obtains the zenith-distance variance σ_z^2 as function of the time variance $\sigma_t^2 = \sigma_1^2 \cdot (\cos \varphi \sin A)^2$, i. e. in the form

$$\sigma_z^2 = \sigma_1^2 + \sigma_2^2 \cdot Q_2, \quad (Q_2 = (\cos \varphi \sin A)^2), \quad (5)$$

on the basis of which one should estimate the variance components σ_1^2 and σ_2^2 .

The results of the variance-component estimation are given in Table 2.

Table 2. The MINQE VC estimates (σ_1^2 , σ_2^2) in VC2 model for three functional adjustment models, FM1, FM2 and FM3; (n – number of observations, u – number of parameters in a linear model).

Funct. Model	n	u	σ_1^2 [″ ²]	σ_2^2 [″ ²]
FM1	1378	17	0.032330	-0.015269
FM2	1396	61	0.020959	-0.003586
FM3	1377	95	0.008975	0.019156

3° "A" Model with Three Components (VC3A).

By studying the dependence of the random-error variance σ_t^2 of the registration time on the individual regressors the present authors establish the

existence of a linear dependence of σ_t on the apparent magnitude of a star. For this reason in Eq. (5) another variance component is introduced so that for σ_z^2 one obtains a model with three components

$$\sigma_z^2 = \sigma_1^2 + \sigma_2^2 \cdot Q_2 + \sigma_3^2 \cdot Q_3. \quad (6)$$

where

$$Q_2 = (\cos \varphi \sin A)^2 \quad \text{and} \quad Q_3 = m_v^2. \quad (7)$$

The MINQE estimates of the variance components σ_1^2 , σ_2^2 and σ_3^2 are given in Table 3.

Table 3. The MINQE VC estimates in VC3A model, σ_1^2 , σ_2^2 , σ_3^2 in Eq. (6) for three functional adjustment models, FM1, FM2 and FM3; (n – number of observations, u – number of parameters in a linear model).

Funct. Model	n	u	σ_1^2 [″ ²]	σ_2^2 [″ ²]	σ_3^2 [″ ²]
FM1	1378	17	0.025385	-0.015731	0.0003646
FM2	1396	61	0.011270	-0.002225	0.0004710
FM3	1377	95	0.004828	0.019495	0.0002073

4° "B" Model with Three Variance Components (VC3B).

It is well known that the higher air temperature is, the more significant is the scattering of rays during their propagation through the lower atmospheric layers. For this reason the present authors include in the variance-component model the variance component proportional to the air temperature T . Now the VC model 3B is

$$\sigma_z^2 = \sigma_2^2 \cdot Q_2 + \sigma_3^2 \cdot Q_3 + \sigma_4^2 \cdot Q_4, \quad (8)$$

where

$$Q_2 = (\cos \varphi \sin A)^2, \quad Q_3 = m_v^2 \quad \text{and} \quad Q_4 = T. \quad (9)$$

The MINQE estimates for VC, (σ_2^2 , σ_3^2 , σ_4^2) are given in Table 4.

Table 4. The MINQE VC estimates in VC3B model, (σ_2^2 , σ_3^2 , σ_4^2) for three functional adjustment models FM1, FM2 and FM3; (n – number of observations, u – number of parameters in a linear model).

Funct. Model	n	u	σ_2^2 [″ ²]	σ_3^2 [″ ²]	σ_4^2 [″ ²]
FM1	1378	17	0.008874	0.0005057	0.0007374
FM2	1396	61	0.011325	0.0005706	0.0002396
FM3	1377	95	0.024344	0.0002524	0.0001180

5. CONCLUDING CONSIDERATIONS

The VC model (4) belongs to the so-called group of models *a priori* guaranteeing positive estimates for variance components. The other VC models, VC2, VC3A and VC3B guarantee positive estimates for variance components only provided Functional Model FM3 is used. According to the present authors the influences of factors affecting the observations are well described by this model, i.e. they consider it is *adequate* one. Therefore, the VC model will be chosen for the case of using Functional Model FM3. The measure used for this choice is the mean weight \bar{P} (mean value taken from all weights), i. e. the ratio \bar{P}_{max}/\bar{P} . These results are given in Table 5.

Table 5.

VC Model	\bar{P}	\bar{P}_{max}/\bar{P}
KDW	61.9584	1.0253
KD2	62.5568	1.0155
KD3A	63.1317	1.0062
KD3B	63.5247	1.0000

On the basis of the mean weights \bar{P} , presented in Table 5. it is concluded that the best weights

are obtained by using the VC3B variance-component model. Nevertheless, it should be said that no model yields significantly lower variance components, i. e. significantly better ones are not obtained (in the sense of a higher observation weight) so that, from the applicative aspect, all four weight models can be used on equal terms.

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АНАЛИЗА КОМПОНЕНТИ ДИСПЕРЗИЈА ПРИ ИЗРАВНАЊУ МРЕЖЕ ЛОНГИТУДА

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Оригинални научни рад

Проблем тежина у задатку изравнања мрежа лонгитуда до сада није разрешен због непознавања структуре дисперзија опажања. При изравнању дела Европске мреже лонгитуда (ELN), које је рађено у оквиру пројекта укључења Београда у ову мрежу,

анализирани су нови модели за одређивање компоненти дисперзија опажања. Овим моделима боље је описана структура дисперзија, тако да су тежине ближе њиховим правим вредностима од до сада коришћених.